

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Friday 12 June 2015 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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[illegible]

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer for each question in the space following the question.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 9 questions in this question paper. The total mark for this paper is 75.

There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

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Turn over

PEARSON

1. Given that

$$\tan \theta^\circ = p, \text{ where } p \text{ is a constant, } p \neq \pm 1$$

use standard trigonometric identities, to find in terms of p ,

(a) $\tan 2\theta^\circ$ (2)

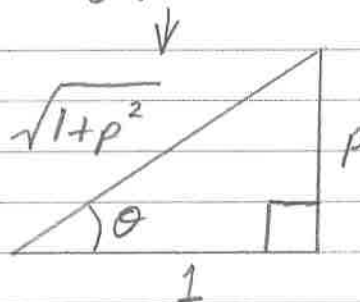
(b) $\cos \theta^\circ$ (2)

(c) $\cot(\theta - 45)^\circ$

Write each answer in its simplest form.

Triangle is a
useful double check. (2)
e.g. for (b)

$$\begin{aligned} \text{a) } \tan 2\theta &\equiv \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2p}{1 - p^2} \end{aligned}$$



$$\text{b) } \sec^2 \theta = 1 + \tan^2 \theta$$

$$\cos^2 \theta = \frac{1}{1 + \tan^2 \theta}$$

$$\cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}} = \frac{1}{\sqrt{1 + p^2}}$$

$$\text{c) } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(\theta - 45^\circ) = \frac{\tan \theta - \tan 45^\circ}{1 + \tan \theta \tan 45^\circ}$$

$$\cot(\theta - 45^\circ) = \frac{1 + \tan \theta (1)}{\tan \theta - 1} = \frac{1 + p}{p - 1}$$



2. Given that

$$f(x) = 2e^x - 5, \quad x \in \mathbb{R}$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

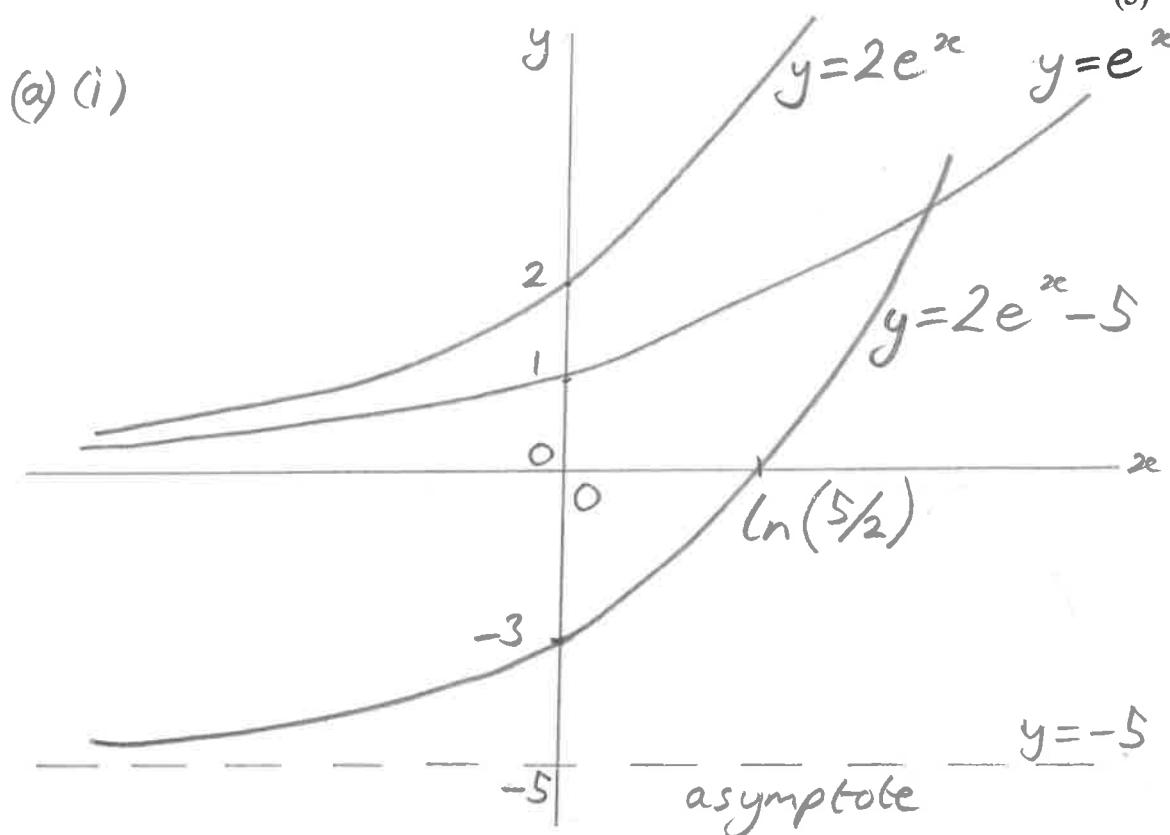
(6)

(b) Deduce the set of values of x for which $f(x) = |f(x)|$

(1)

(c) Find the exact solutions of the equation $|f(x)| = 2$

(3)

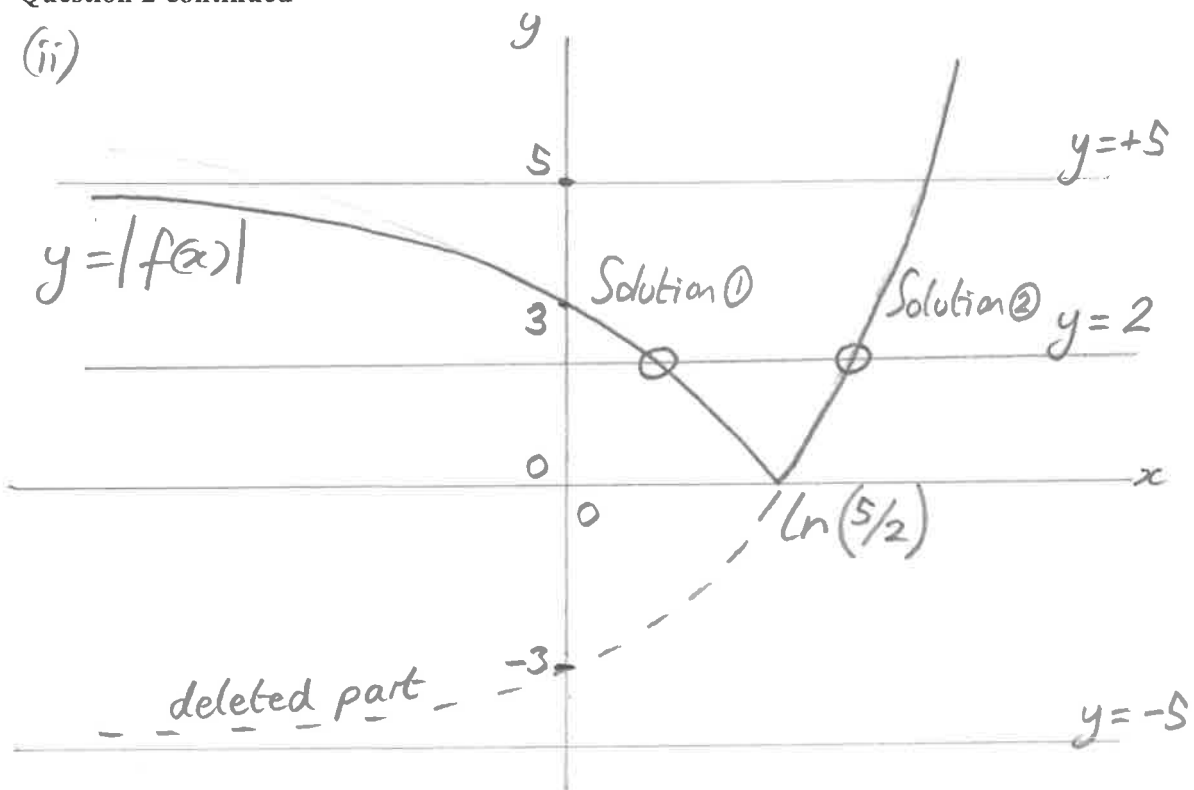


When $y = 2e^{2x} - 5 = 0$
 $e^{2x} = 5/2$
 $x = \ln(5/2)$



Question 2 continued

(ii)



$$b) f(x) = |f(x)| \quad \text{for } x \geq \ln(5/2)$$

$$c) |f(x)| = 2$$

Solution ①

Solution ②

$$|f(x)| = -f(x)$$

$$2e^x - 5 = 2$$

$$-(2e^x - 5) = +2$$

$$2e^x = 7$$

$$-2e^x + 5 = 2$$

$$e^x = 7/2$$

$$-2e^x = -3$$

$$\underline{x = \ln(7/2)}$$

$$\underline{x = \ln(3/2)} \quad \checkmark$$



3.

$$g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta$$

Given that $g(\theta) = R \cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the exact value of R and the value of α to 2 decimal places.

(3)

(b) Hence solve, for $-90^\circ < \theta < 90^\circ$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k .

(2)

$$a) \quad 4 \cos 2\theta + 2 \sin 2\theta = R \cos(2\theta - \alpha)$$

$$4 \cos 2\theta + 2 \sin 2\theta = R \cos 2\theta \cos \alpha + R \sin 2\theta \sin \alpha$$

Comparing terms:

$$4^2 + 2^2 = R^2$$

$$R = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$$

$$R \cos \alpha = 4$$

$$\cos \alpha = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\Rightarrow \alpha = 26.57^\circ \quad (2 \text{ d.p.})$$

$$\therefore 4 \cos 2\theta + 2 \sin 2\theta = 2\sqrt{5} \cos(2\theta - 26.57^\circ)$$



Question 3 continued

b) Solve $4 \cos 2\theta + 2 \sin 2\theta = 1$

$$2\sqrt{5} \cos(\theta - 26.57^\circ) = 1$$

$$\cos(\theta - 26.57^\circ) = \frac{1}{2\sqrt{5}}$$

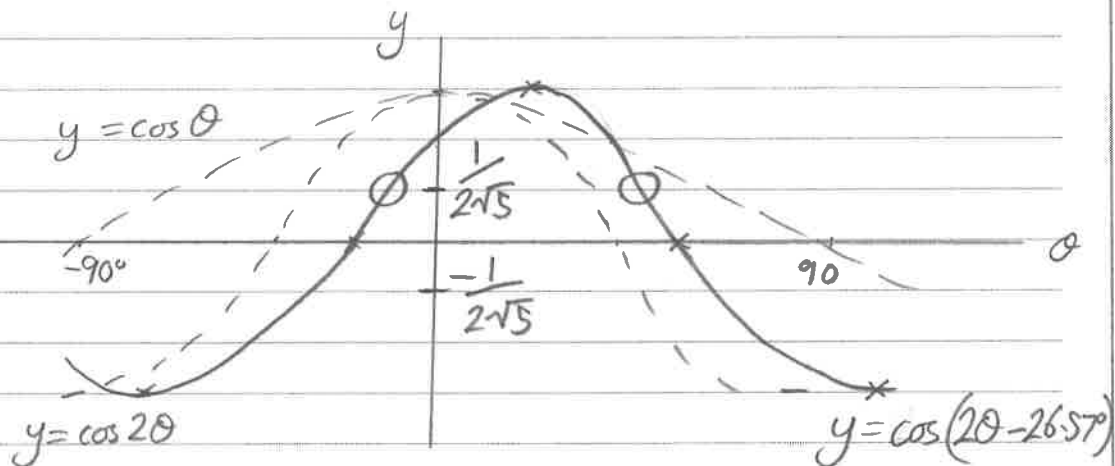
$$2\theta - 26.57^\circ = -77.1^\circ, +77.1^\circ$$

$$2\theta = -50.53^\circ, 103.67^\circ$$

$$\theta = -25.3^\circ, 51.8^\circ$$

Original range: $-90^\circ < x < 90^\circ$

New range: $(-180^\circ - 26.57^\circ) < (2x - 26.57^\circ) < (180^\circ - 26.57^\circ)$



c) If $g(\theta) = k$ has no solutions

Range: $k < -\frac{1}{2\sqrt{5}}$, $k > \frac{1}{2\sqrt{5}}$ ✓ Where $\frac{1}{2\sqrt{5}} = \frac{1}{\sqrt{20}}$



4. Water is being heated in an electric kettle. The temperature, $\theta^\circ\text{C}$, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \quad 0 \leq t \leq T$$

- (a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70°C when $t = 40$,

- (b) find the exact value of λ , giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

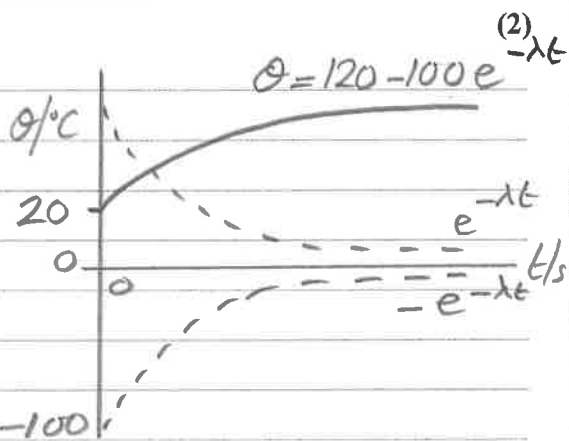
(4)

When $t = T$, the temperature of the water reaches 100°C and the kettle switches off.

- (c) Calculate the value of T to the nearest whole number.

a) See sketch graph

When $t = 0$, $\theta = 120 - 100e^0$
 $\theta = 20^\circ\text{C}$



b) $\theta = 70^\circ\text{C}$ when $t = 40$ s

$$\therefore 70 = 120 - 100e^{-\lambda 40}$$

$$70 - 120 = -100e^{-\lambda 40}$$

$$e^{-40\lambda} = 50/100$$

$$-40\lambda = \ln\left(\frac{1}{2}\right)$$

$$+40\lambda = \ln(2)$$

$$\lambda = \frac{1}{40} \ln(2)$$

c) $t = T$ when $\theta = 100^\circ\text{C}$

$$\therefore \theta = 120 - 100e^{-\lambda T}$$

$$100 = 120 - 100e^{-\frac{\ln 2}{40} T}$$

c) (cont'd)

$$-20 = -100e^{-\frac{\ln 2}{40} T}$$

$$-\frac{\ln 2}{40} T = \ln\left(\frac{20}{100}\right)$$

$$+\frac{\ln 2}{40} T = \ln(5)$$

$$T = \frac{40 \ln(5)}{\ln(2)}$$

$$T = 92.877... \approx 93 \text{ s}$$



5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

- (a) find the exact value of p .

(1)

The tangent to the curve at P cuts the y -axis at the point A .

- (b) Use calculus to find the coordinates of A .

(6)

a) $x = (4y - \sin 2y)^2$

At $P\left(p, \frac{\pi}{2}\right)$:

$$p = \left(4 \frac{\pi}{2} - \sin 2 \frac{\pi}{2}\right)^2$$

$$p = (2\pi - \sin \pi)^2$$

$$p = (2\pi - 0)^2$$

$$\underline{p = 4\pi^2}$$

b) Tangent to curve at P

$$x = (4y - \sin 2y)^2$$

$$\sqrt{x} = 4y - \sin 2y$$

$$x^{\frac{1}{2}} = 4y - \sin 2y$$

$$\frac{d}{dx} x^{\frac{1}{2}} = \frac{d}{dx} 4y - \frac{d}{dx} \sin 2y \quad \text{Implicit method C4}$$

$$\frac{1}{2} x^{-\frac{1}{2}} = 4 \frac{dy}{dx} - 2 \cos 2y \frac{dy}{dx}$$



Question 5 continued

$$x^{-\frac{1}{2}} = 2 \frac{dy}{dx} (4 - 2 \cos 2y)$$

$$x^{-\frac{1}{2}} = 4 \frac{dy}{dx} (2 - \cos 2y)$$

$$\frac{dy}{dx} = \frac{1}{4 x^{\frac{1}{2}} (2 - \cos 2y)}$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=4\pi^2 \\ y=\frac{\pi}{2}}} = \frac{1}{4 \sqrt{4\pi^2} (2 - \cos 2 \cdot \frac{\pi}{2})}$$

$$m = \frac{1}{8\pi (2 - -1)} = \frac{1}{24\pi}$$

Equation of tangent at P:

$$y - y_1 = m (x - x_1)$$

$$y - \frac{\pi}{2} = \frac{1}{24\pi} (x - 4\pi^2)$$

$$y = \frac{1}{24\pi} x - \frac{\pi}{6} + \frac{\pi}{2}$$

$$y = \frac{1}{24\pi} x + \frac{\pi}{3}$$

∴ y intercept is $y = \frac{\pi}{3}$



6.

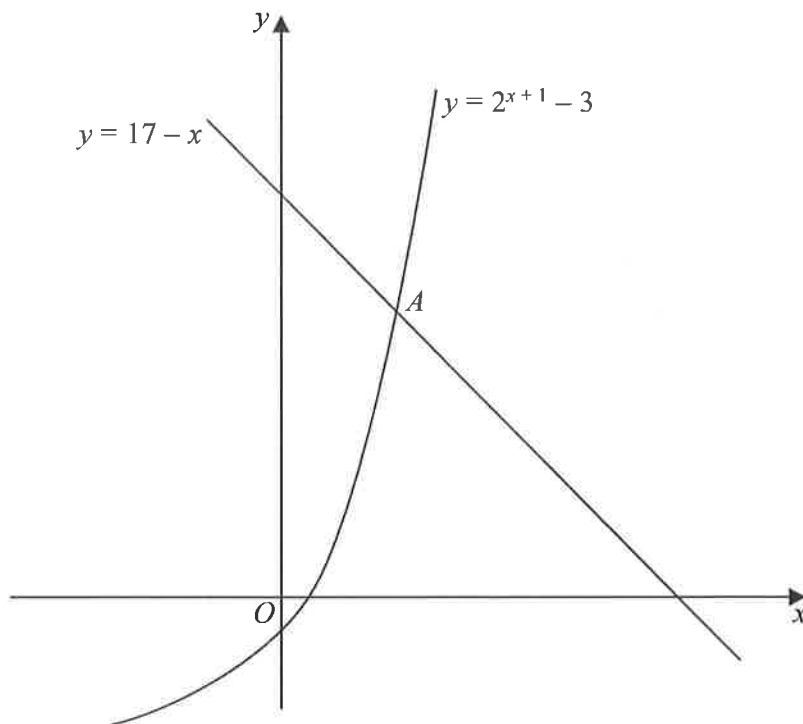
**Figure 1**

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A .

- (a) Show that the x coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1 \quad (3)$$

- (b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \quad x_0 = 3$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

- (c) Use your answer to part (b) to deduce the coordinates of the point A , giving your answers to one decimal place. (2)



Question 6 continued

Curve C: $y = 2^{x+1} - 3$

Line L: $y = 17 - x$

a) Intersect at A:

$$y = 17 - x = 2^{x+1} - 3$$

$$20 - x = 2^{x+1}$$

$$\ln(20 - x) = (x+1) \ln(2)$$

$$x = \frac{\ln(20 - x)}{\ln(2)} - 1$$

b) Iterative method:

$$x_0 = 3$$

$$x_1 = \frac{\ln(20-3)}{\ln(2)} - 1 = 3.0874628...$$

$$x_2 = \frac{\ln(20-3.087...)}{\ln(2)} - 1 = 3.080021199...$$

$$x_3 = \frac{\ln(20-3.0800...)}{\ln(2)} - 1 = 3.0806558... \\ \approx 3.081 \text{ (3DP)}$$

c) $y = 17 - x$
 $\approx 17 - 3.081$
 $\approx 13.919...$

$y \approx 13.9$ (1D.P)



7.

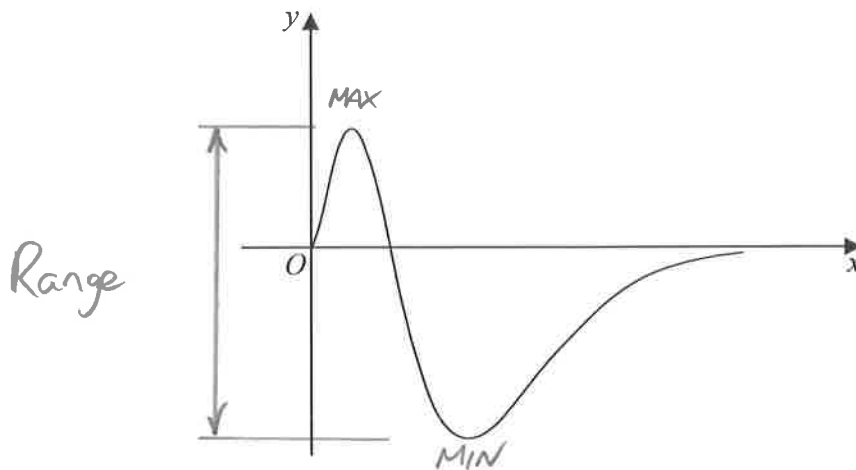


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1-x)e^{-2x}, \quad x \geq 0$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g .

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

$$a) \quad g(x) = x^2(1-x)e^{-2x}$$

$$g(x) = (x^2 - x^3)e^{-2x}$$

$$\text{Let } u = x^2 - x^3 \quad \text{Let } v = e^{-2x}$$

$$\frac{du}{dx} = 2x - 3x^2$$

$$\frac{dv}{dx} = -2e^{-2x}$$

Using product rule:

$$\begin{aligned} \frac{d}{dx}(uv) &= v \frac{du}{dx} + u \frac{dv}{dx} \\ &= e^{-2x}(2x - 3x^2) + (x^2 - x^3)(-2e^{-2x}) \end{aligned}$$



Question 7 continued

$$= e^{-2x} (2x - 3x^2 - 2x^2 + 2x^3)$$

$$g'(x) = (2x^3 - 5x^2 + 2x) e^{-2x} = f(x) e^{-2x}$$

b) Range of $g(x)$ = Max - Min values of y
(ie turning points)

$$\text{When } (2x^3 - 5x^2 + 2x) e^{-2x} = 0$$

$$\text{Either } e^{-2x} = 0 \Rightarrow x = 0 \text{ Reject as } x \geq 0$$

$$\text{Or } x(2x^2 - 5x + 2) = 0 \text{ Reject } x = 0$$

$$\text{Or } 2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$\therefore x = \frac{1}{2} \quad \text{or} \quad x = 2$$

$$y = \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right) e^{-2\left(\frac{1}{2}\right)}$$

$$y = (2)^2 (1 - 2) e^{-2(2)}$$

$$y = \frac{1}{2^3} e^{-1}$$

$$y = -2^2 e^{-4}$$

So range from $-4e^{-4}$ to $\frac{1}{8}e^{-1}$

c) $g(x)$ is many to one function

$g'(x)$ would be one to many so not possible.



8. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \quad A \neq \frac{(2n+1)\pi}{4}, \quad n \in \mathbb{Z} \quad (5)$$

(b) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

$$\text{a) LHS: } \sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$$

$$= \frac{1 + \sin 2A}{\cos 2A}$$

$$= \frac{1 + 2\sin A \cos A}{\cos^2 A - \sin^2 A}$$

$$[\text{Using } \cos^2 A + \sin^2 A = 1] = \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A} = \text{RHS}$$

$$\text{b) If } \sec 2\theta + \tan 2\theta = \frac{1}{2}$$

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{1}{2}$$

$$\cos \theta + \sin \theta = \frac{1}{2} \cos \theta - \frac{1}{2} \sin \theta$$

$$\frac{1}{2} \cos \theta + \frac{3}{2} \sin \theta = 0$$

$$\cos \theta + 3 \sin \theta = 0 \quad (\times 2)$$



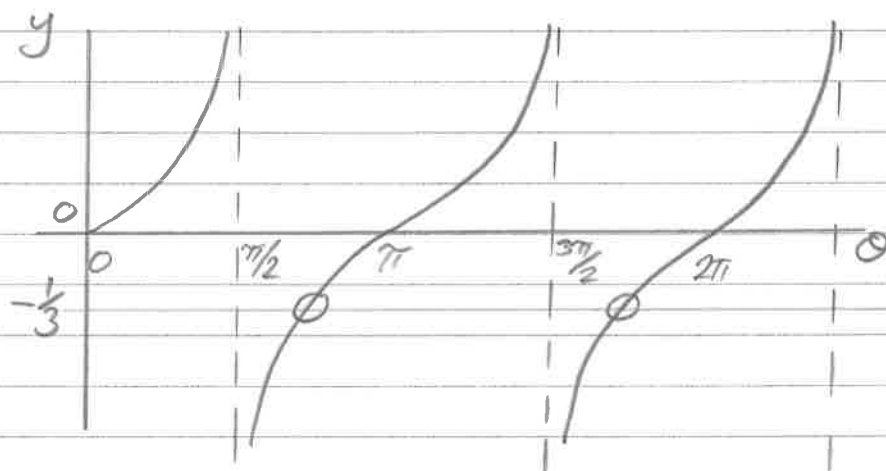
$$\frac{\cos \theta}{\cos \theta} + \frac{3 \sin \theta}{\cos \theta} = 0 \quad (\div \cos \theta)$$

$$1 + 3 \tan \theta = 0$$

$$\tan \theta = -\frac{1}{3}$$

$Q = -0.32175, 2.81984, 5.96143$
 $\uparrow$
 reject

$\theta = 2.820 \text{ rad}, 5.961 \text{ rad}$
(3pp)



9. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \quad x \geq 0$$

(a) show that $f(x) = \frac{x+k}{x-2k}$ (3)

(b) Hence find $f'(x)$, giving your answer in its simplest form. (3)

- (c) State, with a reason, whether $f(x)$ is an increasing or a decreasing function.

Justify your answer.

(2)

$$a) f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}$$

$$= 2 - \frac{(x-5k)(x-k)}{(x-k)(x-2k)}$$

$$= 2 - \frac{(x-5k)}{(x-2k)}$$

$$= \frac{2(x-2k) - (x-5k)}{x-2k}$$

$$= \frac{2x - 4k - x + 5k}{x - 2k}$$

$$f(x) = \frac{x+k}{x-2k}$$



Question 9 continued

$$b) \text{ Let } u = x + k \qquad \text{Let } v = x - 2k$$

$$\frac{du}{dx} = 1 + 0$$

$$\frac{dv}{dx} = 1 - 0$$

Quotient rule :

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(x-2k)(1) - (x+k)(1)}{(x-2k)^2}$$

$$= \frac{x - 2k - x - k}{(x-2k)^2}$$

$$f'(x) = \frac{-3k}{(x-2k)^2}$$

c) If k is negative

Numerator must be +ve $(-x-)$

Denominator must be +ve $(\text{anything})^2$

So $f(x)$ must be +ve so increasing function.



