

Write your name here			
Surname MODEL ANSWERS	Other names Chris		
Pearson	Centre Number	Candidate Number	
Edexcel GCE			
Core Mathematics C3 Advanced			
Tuesday 21 June 2016 – Morning		Paper Reference	
Time: 1 hour 30 minutes		6665/01	
You must have: Mathematical Formulae and Statistical Tables (Pink)			Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. The functions f and g are defined by

$$f : x \rightarrow 7x - 1, \quad x \in \mathbb{R}$$

$$g : x \rightarrow \frac{4}{x-2}, \quad x \neq 2, x \in \mathbb{R}$$

- (a) Solve the equation $fg(x) = x$

(4)

- (b) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$

(1)

$$a) \quad f \circ g(x) = x$$

$$7\left(\frac{4}{x-2}\right) - 1 = x$$

$$\frac{28}{x-2} - 1 = x$$

$$\frac{28 - (x-2)}{x-2} = x$$

$$28 - x + 2 = x(x-2)$$

$$-x + 30 = x^2 - 2x$$

$$x^2 - x - 30 = 0$$

$$(x+5)(x-6) = 0$$

$$\text{Either } \underline{x = -5} \quad \text{Or } \underline{x = +6}$$

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Question 1 continued

$$b) \quad g(a) = f^{-1}(a)$$

$$\frac{4}{a-2} = \frac{a+1}{7}$$

$$28 = a^2 - a - 2$$

$$a^2 - a - 30 = 0$$

This is the same quadratic as in (a)

Therefore largest value of a is +6

Q1

(Total 5 marks)



2.

$$y = \frac{4x}{x^2 + 5}$$

(a) Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form. (4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} < 0$ (3)

a) Using quotient rule

$$\text{Let } u = 4x$$

$$\text{Let } v = x^2 + 5$$

$$\frac{du}{dx} = 4$$

$$\frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(x^2 + 5)4 - 4x(2x)}{(x^2 + 5)^2}$$

$$= \frac{4x^2 + 20 - 8x^2}{(x^2 + 5)^2}$$

$$\frac{dy}{dx} = \frac{20 - 4x^2}{(x^2 + 5)^2}$$

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Question 2 continued

$$b) \quad \frac{dy}{dx} < 0$$

$$\frac{20 - 4x^2}{(x^2 + 5)^2} < 0$$

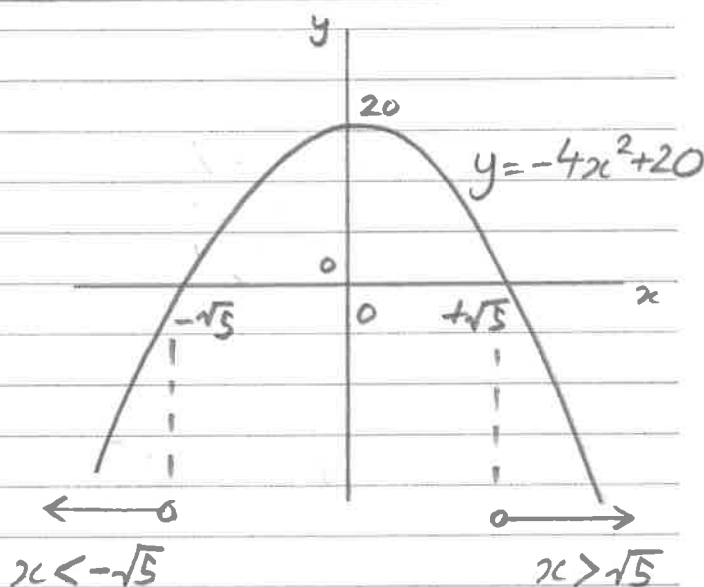
$$20 - 4x^2 < 0$$

$$4x^2 > 20$$

$$x^2 > 5$$

Critical values: $x = \pm \sqrt{5}$

$$x < -\sqrt{5}, \quad x > \sqrt{5}$$



(Total 7 marks)

Q2



P 4 6 7 1 7 A 0 5 3 2

3. (a) Express $2 \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

- (b) Hence solve, for $0 \leq \theta < 360^\circ$,

$$\frac{2}{2 \cos \theta - \sin \theta - 1} = 15$$

Give your answers to one decimal place.

(5)

- (c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2 \cos \theta + \sin \theta - 1} = 15$$

Give your answer to one decimal place.

(2)

$$a) \quad a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$$

$$\text{where } R = \sqrt{a^2 + b^2}$$

$$R = \sqrt{(2)^2 + (-1)^2}$$

$$R = \sqrt{5}$$

Because:

$$R \cos \alpha = a \quad (1)$$

$$R \sin \alpha = b \quad (2)$$

$$\text{where } \alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\alpha = \tan^{-1}\left(\frac{-1}{2}\right)$$

$$\alpha = -26.57^\circ \quad (2 \text{ d.p.})$$

$$\begin{aligned} \Rightarrow 2 \cos \theta - \sin \theta &= \sqrt{5} \cos(\theta - -26.57^\circ) \\ &= \sqrt{5} \cos(\theta + 26.57^\circ) \end{aligned}$$



Question 3 continued

$$b) \quad \frac{2}{2 \cos \theta - \sin \theta - 1} = \frac{15}{1}$$

$$\frac{2 \cos \theta - \sin \theta - 1}{2} = \frac{1}{15}$$

$$2 \cos \theta - \sin \theta = \frac{2}{15} + 1$$

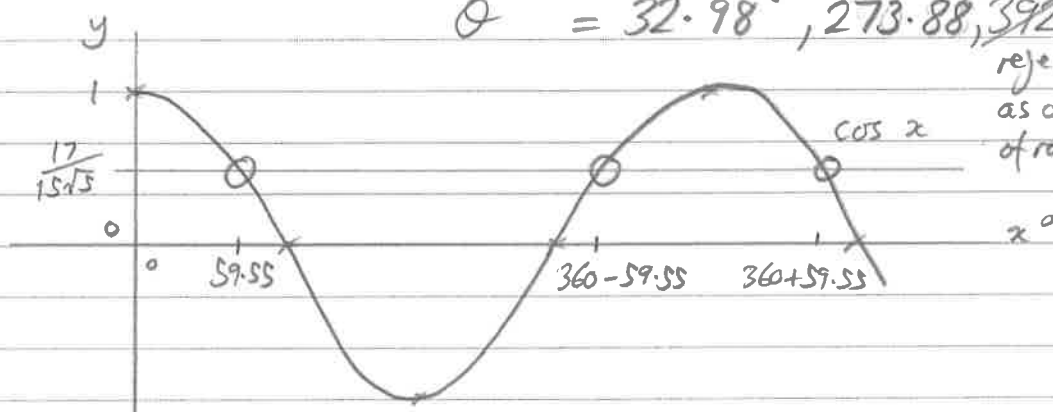
$$\sqrt{5} \cos(\theta + 26.57^\circ) = \frac{17}{15} \quad \text{using (a)}$$

$$\theta + 26.57^\circ = \cos^{-1} \frac{17}{15\sqrt{5}}$$

$$\theta + 26.57^\circ = 59.55^\circ, 300.45^\circ, 419.55^\circ, \dots$$

$$\theta = 32.98^\circ, 273.88^\circ, 392.98^\circ, \dots$$

reject
as out
of range



$$\underline{\theta = 33.0^\circ, 273.9^\circ \text{ (10.P.)}} \quad \checkmark$$



Question 3 continued

$$c) \frac{2}{2 \cos \theta + \sin \theta - 1} = 15$$

↑
sign changed
to positive

$$\Rightarrow \sqrt{5} (\cos \theta - 26.57^\circ) = \frac{17}{15}$$

$$\Rightarrow \theta - 26.57^\circ = 59.55^\circ, 300.45^\circ, \dots$$

$$\underline{\theta = 86.1^\circ, \dots (1 \text{ d.p.})}$$

↑ smallest positive
answer.

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Question 3 continued

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(Total 10 marks)

Q3



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4.

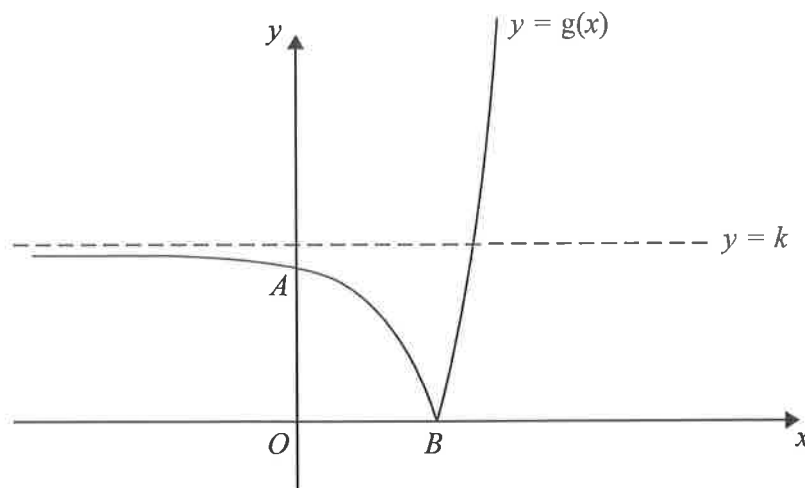


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = g(x)$, where

$$g(x) = |4e^{2x} - 25|, \quad x \in \mathbb{R}$$

The curve cuts the y -axis at the point A and meets the x -axis at the point B . The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1

(a) Find, giving each answer in its simplest form,

- (i) the y coordinate of the point A ,
- (ii) the exact x coordinate of the point B ,
- (iii) the value of the constant k .

(5)

The equation $g(x) = 2x + 43$ has a positive root at $x = \alpha$

(b) Show that α is a solution of $x = \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)$

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2} \ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α

(c) Taking $x_0 = 1.4$ find the values of x_1 and x_2
Give each answer to 4 decimal places.

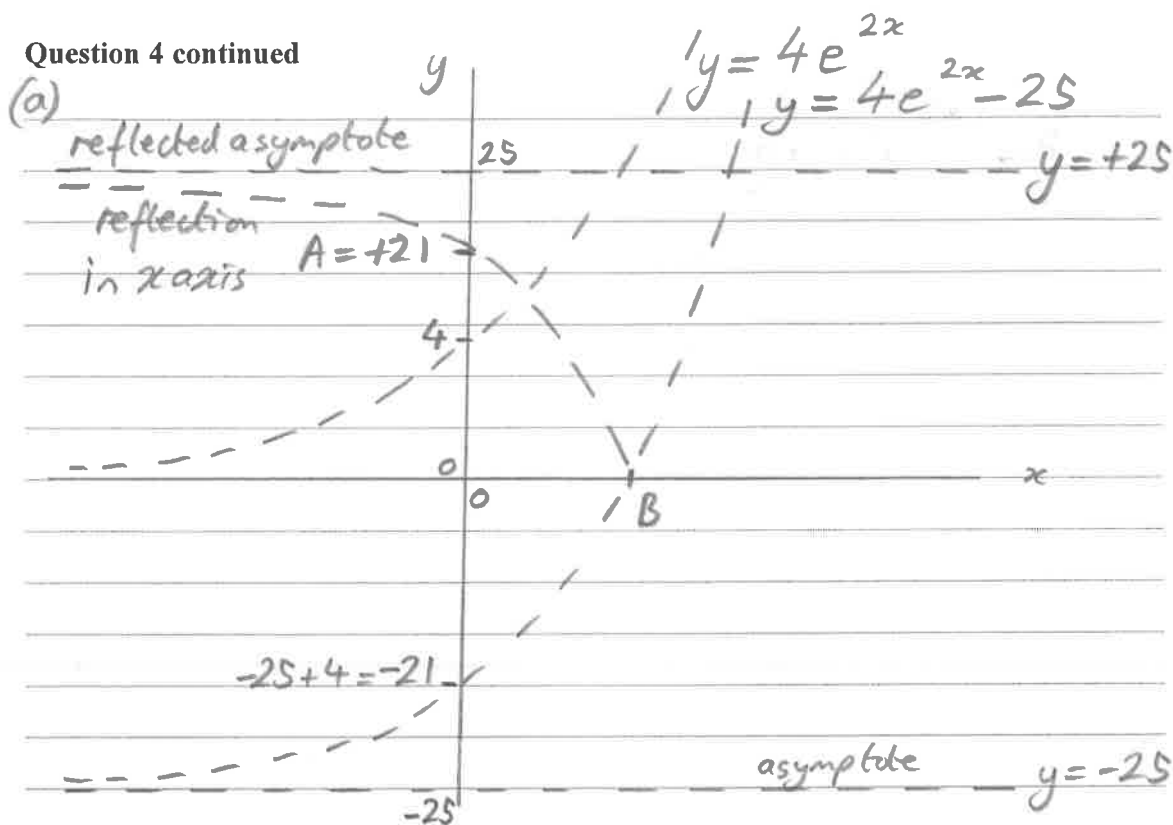
(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)



Question 4 continued



(i) See sketch graph, y coordinate of A is 21

(ii) B: $y = 4e^{2x} - 25 = 0$

$$e^{2x} = \frac{25}{4}$$

$$2x = \ln \frac{25}{4}$$

$$x = \frac{1}{2} \ln \frac{25}{4}$$

$$x = \ln \left(\frac{25}{4} \right)^{\frac{1}{2}} \text{ using log rules}$$

$$x = \ln \frac{5}{2}$$

(iii) See sketch graph, $k = 25$



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Question 4 continued

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Question 4 continued

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(Total 11 marks)

Q4



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5. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leq x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

- (ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y .

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

(i) $y = e^{3x} \cos 4x$

Let $u = e^{3x}$

Let $v = \cos 4x$

$\frac{du}{dx} = 3e^{3x}$ (implicit rule)

$\frac{dv}{dx} = -4 \sin 4x$

Product rule:

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$= \cos 4x (3e^{3x}) + e^{3x} (-4 \sin 4x)$$

$$\frac{dy}{dx} = e^{3x} (3 \cos 4x - 4 \sin 4x)$$

Turning point when $\frac{dy}{dx} = 0$

Either $e^{3x} = 0$ Or $3 \cos 4x - 4 \sin 4x = 0$

reject



Question 5 continued

$$(ii) \quad x = \sin^2 2y$$

$$x = (\sin 2y)(\sin 2y)$$

$$\text{Let } u = \sin 2y \quad x = u^2$$

$$\frac{du}{dy} = 2 \cos 2y$$

$$\frac{dx}{du} = 2u$$

Chain rule:

$$\frac{dx}{dy} = \frac{dx}{du} \times \frac{du}{dy}$$

$$= 2u \times 2 \cos 2y$$

$$= 2 \sin 2y \times 2 \cos 2y$$

$$\frac{dx}{dy} = 4 \sin 2y \cos 2y$$

$$\frac{dy}{dx} = \frac{1}{4 \sin 2y \cos 2y}$$

Using $\sin 2A = 2 \sin A \cos A$

$$= \frac{1}{2} \times \frac{1}{\sin 4y}$$

$$\frac{dy}{dx} = \frac{1}{2} \operatorname{cosec} 4y$$

$$p = \frac{1}{2}$$

$$q = 4$$

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Question 5 continued

$$3 \cos 4x = 4 \sin 4x$$

$$\frac{\sin 4x}{\cos 4x} = \frac{3}{4}$$

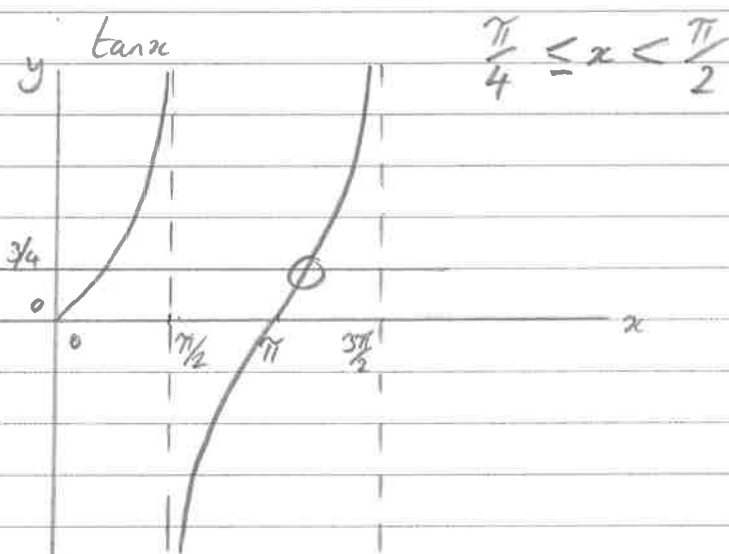
$$\tan 4x = \frac{3}{4}$$

$$x = \frac{1}{4} \tan^{-1} \frac{3}{4}$$

$$x = \frac{1}{4} (0.6435 \text{ rad}, 3.785 \text{ rad}, \dots)$$

$$x \approx 0.1609 \text{ rad}, \underline{0.9463 \text{ rad}}$$

↑
reject,
out of range



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Q5

(Total 10 marks)



6. $f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \quad x > 2, x \in \mathbb{R}$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B .

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$

(5)

a) Algebraic long division:

$$\begin{array}{r} x^4 + x^3 - 3x^2 + 7x - 6 \\ \underline{x^2 + x - 6} \\ x^2 + x - 6 \mid x^4 + x^3 - 3x^2 + 7x - 6 \quad \leftarrow \text{polynomial} \\ \quad \quad \quad \uparrow \quad \quad \quad \downarrow \quad \quad \downarrow \\ \text{divisor} \quad - (x^4 + x^3 - 6x^2) \quad \quad \quad \downarrow \quad \quad \downarrow \\ \quad \quad \quad 0 + 0 + 3x^2 + 7x - 6 \\ \quad \quad \quad \underline{-(3x^2 + 3x - 18)} \\ \quad \quad \quad 4x + 12 \quad \text{remainder.} \end{array}$$

$$\begin{aligned} \therefore \frac{x^4 + x^3 - 3x^2 + 7x - 6}{(x+3)(x-2)} &= x^2 + 3 + \frac{4x+12}{(x+3)(x-2)} \\ &= x^2 + 3 + \frac{4(x+3)}{(x+3)(x-2)} \\ &= x^2 + 3 + \frac{4}{x-2} \end{aligned}$$



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Question 6 continued

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Question 6 continued

$$b) f(x) = y = x^2 + 3 + \frac{4}{x-2}$$

To differentiate $y = \frac{4}{x-2}$

Let $u = x-2$

$$\frac{du}{dx} = 1$$

Let $y = 4u^{-1}$

$$\frac{dy}{du} = -4u^{-2}$$

$$\therefore \frac{dy}{dx} = -\frac{4}{(x-2)^2}$$

$$\therefore f'(x) = 2x + 0 - \frac{4}{(x-2)^2} \quad \text{gradient of curve C}$$

Gradient of tangent at point P, $x=3$:

$$f'(3) = 2(3) - \frac{4}{(3-2)^2} = 6 - 4 = 2$$

$$\therefore m_T = 2$$

Normal: $m_N = -\frac{1}{m_T} = -\frac{1}{2}$

P: $f(3) = y = (3)^2 + 3 + \frac{4}{(3)-2}$

$$y = 9 + 3 + 4 = 16$$

P: $(3, 16)$



Question 6 continued

Equation of normal to curve C at P(3, 16)

$$y - y_1 = m_n (x - x_1)$$

$$y - 16 = -\frac{1}{2} (x - 3)$$

$$y = -\frac{1}{2}x + \frac{3}{2} + 16$$

Normal: $y = -\frac{1}{2}x + \frac{35}{2}$

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Q6

(Total 9 marks)



7. (a) For $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arcsin x \quad -1 \leq x \leq 1$$

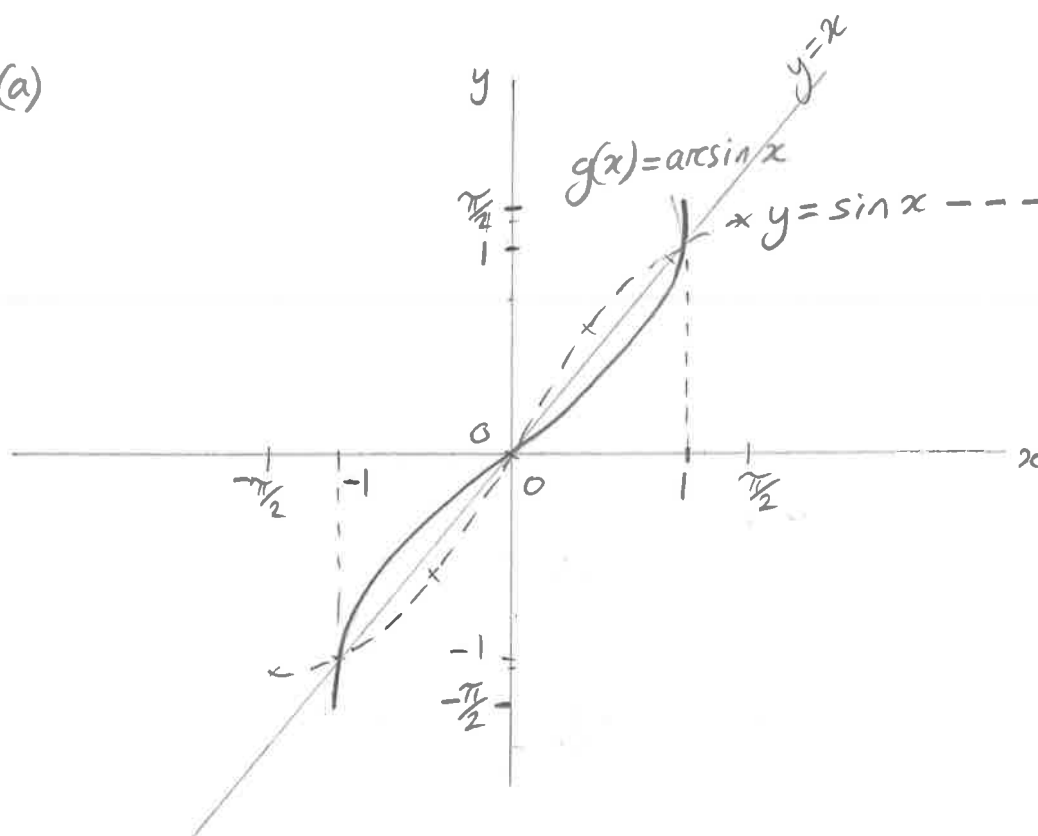
(2)

- (b) Find the exact value of x for which

$$3g(x + 1) + \pi = 0$$

(3)

(a)



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Question 7 continued

$$b) \quad g(x) = \arcsin x$$

$$3g(x+1) + \pi = 0$$

$$3\arcsin(x+1) + \pi = 0$$

$$3\arcsin(x+1) = -\pi$$

$$\arcsin(x+1) = -\frac{\pi}{3}$$

$$x+1 = \sin\left(-\frac{\pi}{3}\right)$$

$$x+1 = -\frac{\sqrt{3}}{2}$$

$$x = -\frac{\sqrt{3}}{2} - 1$$

$$x = \underline{\underline{-\frac{\sqrt{3}-2}{2}}}$$

Q7

(Total 5 marks)



8. (a) Prove that

$$2 \cot 2x + \tan x \equiv \cot x \quad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

a)

$$\text{LHS} = 2 \cot 2x + \tan x$$

$$= \frac{2}{\tan 2x} + \tan x$$

$$= \frac{2(1 - \tan^2 x)}{2 \tan x} + \tan x$$

Using identity
 $\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$

$$= \frac{1 - \cancel{\tan^2 x} + \cancel{\tan^2 x}}{\tan x}$$

$$= \frac{1}{\tan x}$$

$$\text{LHS} = \cot x = \text{RHS}$$

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Question 8 continued

b)

$$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2$$

$$3(2 \cot 2x + \tan x) = \operatorname{cosec}^2 x - 2$$

$$3(\cot x) = \operatorname{cosec}^2 x - 2$$

$$3 \cot x = 1 + \cot^2 x - 2$$

$$\cot^2 x - 3 \cot x - 1 = 0$$

Does not factorise, so use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cot x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

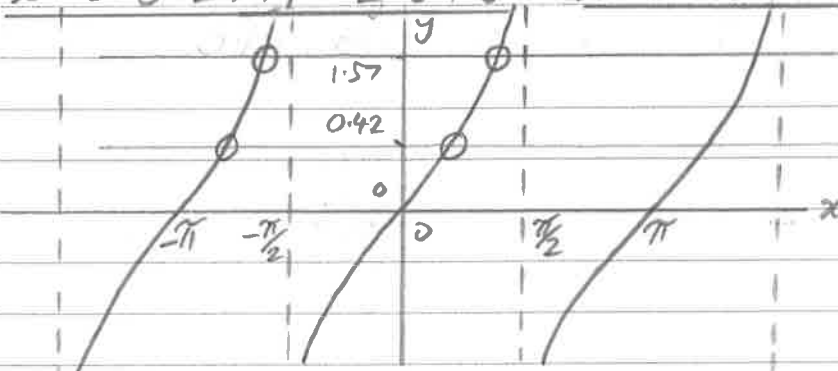
$$\cot x = \frac{3 \pm \sqrt{13}}{2}$$

Either

$$x = \tan^{-1} \frac{2}{3 + \sqrt{13}} \quad \text{or} \quad x = \tan^{-1} \frac{2}{3 - \sqrt{13}}$$

$$x = 0.294, -\pi + 0.294 \quad x = -1.277, \pi - 1.277$$

$$x = 0.294, -2.848 \quad x = -1.277, 1.865$$



Question 8 continued

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Question 8 continued

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(Total 10 marks)

Q8



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9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

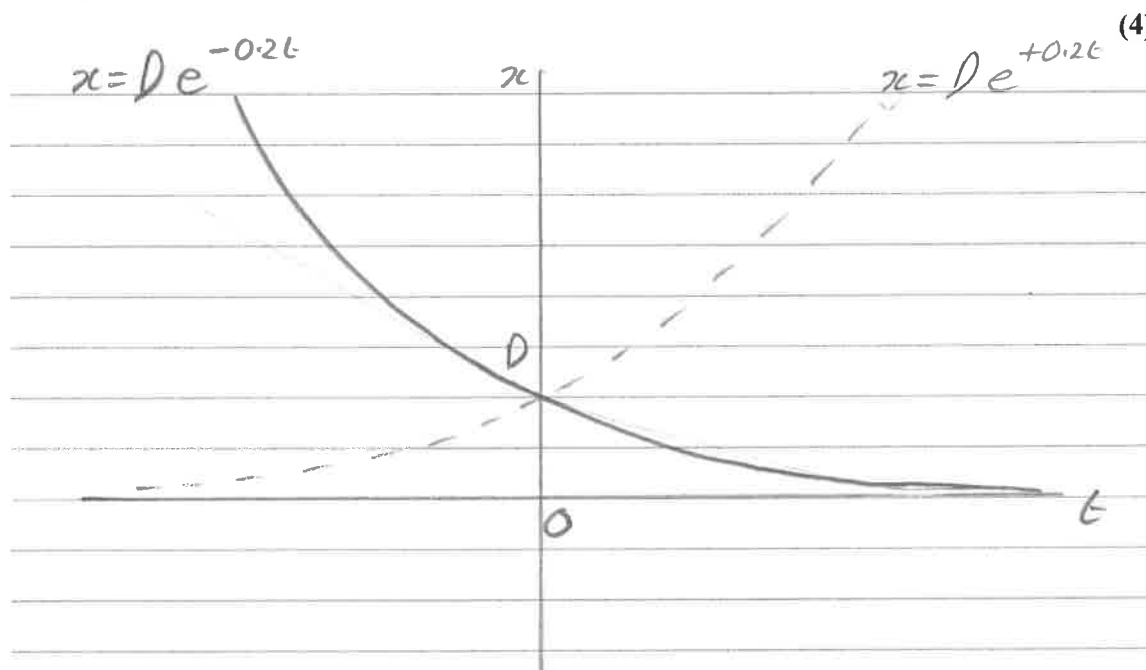
- (a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places. (2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

- (b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places. (2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

- (c) Show that $T = a \ln \left(b + \frac{b}{e} \right)$, where a and b are integers to be determined. (4)



Question 9 continued

$$x_{\text{TOTAL}} = x_1 + x_2$$

$$= x(5+T) + x(T)$$

$$7.5 = 15e^{-0.2(5+T)} + 15e^{-0.2T}$$

$$\frac{1}{2} = e^{-0.2(5+T)} + e^{-0.2T}$$

$$\frac{1}{2} = e^{-0.2T} (1 + e^{-1})$$

$$\ln \frac{1}{2} - \ln(1 + \frac{1}{e}) = -0.2T$$

$$T = -5 \left(\ln \frac{1}{2} - \ln(1 + \frac{1}{e}) \right)$$

$$T = +5 \left(\ln 2 + \ln(1 + \frac{1}{e}) \right)$$

$$T = 5 \left(\ln \left(2 + \frac{2}{e} \right) \right)$$

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Question 9 continued

$$x = D e^{-0.2t} \quad (1)$$

$$\ln x = \ln D - 0.2t \quad (2)$$

a) Dis the starting amount of 15 mg

$$\therefore x = 15 e^{-0.2t}$$

$$x(4) = 15 e^{-0.2 \times 4}$$

$$x = 6.73993$$

$$\underline{x \approx 6.740 \text{ mg}} \quad (3 \text{ D.P.})$$

b) First dose after $5+2=7$ hours.

$$x(7) = 15 e^{-0.2 \times 7} = 3.699 \text{ mg.}$$

Second dose after 2 hours.

$$x(2) = 15 e^{-0.2 \times 2} = 10.055 \text{ mg.}$$

$$\text{Total } x = 3.699 + 10.055$$

$$\underline{x = 13.754 \text{ mg}} \quad (3 \text{ D.P.})$$



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Question 9 continued

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Question 9 continued

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Q9

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

END

