PMT

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature MODEL ANSWERS	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point (3, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

a) $x^3 + 2xy - x - y^3 - 20 = 0$

$\frac{d}{dx}(x^3) + 2x\frac{d}{dx}(y) + y\frac{d}{dx}(2x) - \frac{d}{dx}(x) - \frac{d}{dx}(y^3) - \frac{d}{dx}(20)$

$3x^2 + 2x\frac{dy}{dx} + y \times 2 - 1 - 3y^2\frac{dy}{dx} - 0 = 0$

$\frac{dy}{dx}(2x - 3y^2) + 3x^2 + 2y - 1 = 0$

$\frac{dy}{dx} = \frac{1 - 3x^2 - 2y}{2x - 3y^2}$ ✓

b) At C(3, -2)

$\left.\frac{dy}{dx}\right|_{\substack{x=3\\y=-2}} = \frac{1 - 3(3)^2 - 2(-2)}{2(3) - 3(-2)^2} = \frac{1 - 27 + 4}{6 - 12}$

$= \frac{-22}{-6} = \frac{11}{3} = m_T$ tangent gradient

Using, $y - y_1 = m(x - x_1)$

$y - (-2) = \frac{11}{3}(x - (3))$

$y + 2 - \frac{11x}{3} + 11 = 0 \Rightarrow 11x - 3y - 39 = 0$ ✓

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2. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(a) find the value of the constant k,

(2)

(b) find the value of the constant A, giving your answer in its simplest form.

(3)

a)

$$(1+ax)^n = 1 + n(ax) + \frac{n(n-1)(ax)^2}{2!} + \frac{n(n-1)(n-2)(ax)^3}{3!} + \ldots$$

$$(1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)(kx)^2}{2!} + \ldots$$

$$= 1 - 6x + A\,x^2 + \ldots$$

Comparing terms : $-6 = -4k$ (Coefficients of x)

$\underline{k = 3/2}$ ✓

b) Comparing coefficients of x^2

$$A = \frac{(-4)(-4-1)k^2}{2!}$$

$$= \frac{20}{2}\left(\frac{3}{2}\right)^2$$

$$= \frac{90}{4}$$

$$= \frac{45}{2}$$

$\underline{A = 22.5}$ ✓

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Question 2 continued

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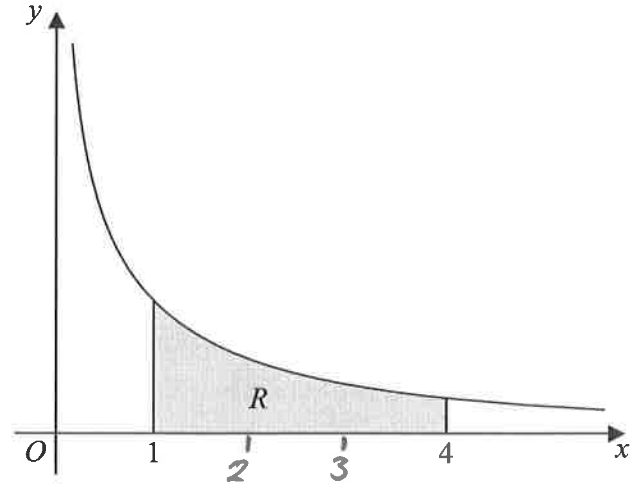
3.



y_0 y_1 **Figure 1** y_2 y_3

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}, \quad x > 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$

x	1	2	3	4
y	1.42857	0.90326	0·68212	0.55556
	y_0	y_1	y_2	y_3

$h = 1$ ✓

(a) Complete the table above by giving the missing value of y to 5 decimal places.
(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.
(3)

(c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of R.
(1)

(d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}} \, dx$$

(6)

Trapezium rule

Area $= \frac{1}{2} h[y_0 + 2(y_1 + y_2) + y_3]$

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Question 3 continued

$$\text{Area} = \tfrac{1}{2}(1)\left[1.42857 + 2(0.90326 + 0.68212) + 0.55556\right]$$

$$= 2.577445 \simeq \underline{2.5774} \quad (4\text{D.P.})$$

✓

c)

concave curve

The area of the trapezium is greater than the area under the curve - it is an over-estimate of area.

✓

d) $$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$

Let $u = \sqrt{x} \Rightarrow x = u^2 \Rightarrow dx = 2u\,du$

If $x = 4$, $u = 2$ If $x = 1$, $u = 1$ (Change limits)

$$I = 10\int_{u=1}^{u=2} \frac{1}{2u^2 + 5u}\,dx = 10\int_1^2 \frac{1 \times 2u}{u(2u+5)}\,du$$

$$= 20\int_1^2 \frac{1}{2u+5}\,du = 20 \times \tfrac{1}{2}\left[\ln(2u+5)\right]_1^2$$

$$= 10(\ln 9 - \ln 7)$$

$$\underline{I = 10\ln\left(\tfrac{9}{7}\right)}$$

✓

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4.

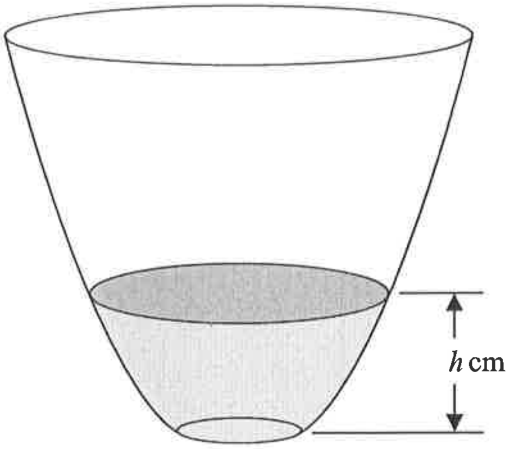

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water $V\text{ cm}^3$ is given by

$$V = 4\pi h(h+4), \qquad 0 \leqslant h \leqslant 25$$

Water flows into the vase at a constant rate of $80\pi\text{ cm}^3\text{s}^{-1}$

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$

(5)

$\frac{dV}{dt} = 80\pi \text{ cm}^3\text{s}^{-1}$

$\Rightarrow V = \int 80\pi \, dt = 80\pi t + c$

V=0 when h=0 when t=0 so c=0

$\Rightarrow 80\pi t = 4\pi(h^2 + 4h)$

$t = \frac{1}{20}(h^2 + 4h)$

$\frac{dt}{dh} = \frac{1}{20}(2h + 4)$

$\Rightarrow \frac{dh}{dt} = \frac{20}{2h+4}$

$\left.\frac{dh}{dt}\right|_{h=6} = \frac{20}{2(6)+4} = \frac{5}{4} = 1.25 \text{ cms}^{-1}$

✓

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5.

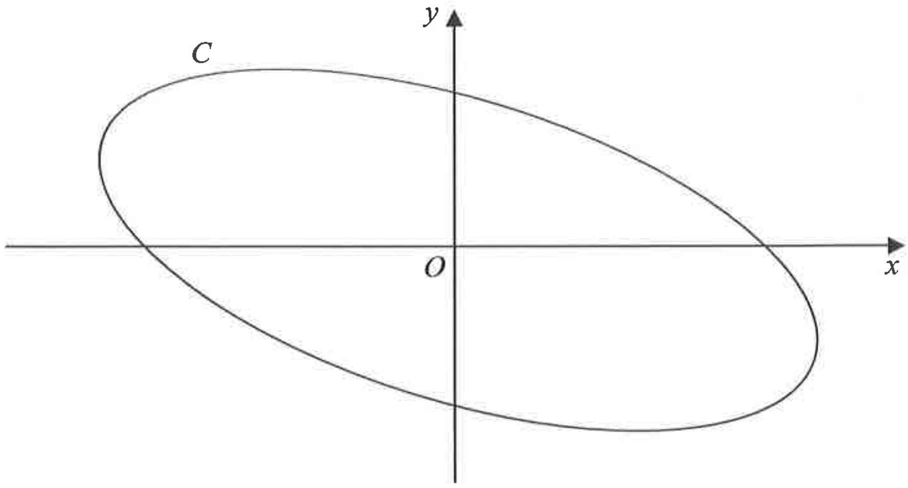

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

a) $x = 4\cos(t + \pi/6)$

$= 4(\cos t \cos \pi/6 - \sin t \sin \pi/6)$

$= 4\cos t(\sqrt{3}/2) - 4\sin t\,(\tfrac{1}{2})$

$\therefore x + y = 2\sqrt{3}\cos t - \cancel{2\sin t} + \cancel{2\sin t}$

$= 2\sqrt{3}\cos t$

✓

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Question 5 continued

b) $(x+y)^2 + ay^2 = b$

$\cos t = \frac{x+y}{2\sqrt{3}}$

$\sin t = \frac{y}{2}$

Using,

$\sin^2 t + \cos^2 t \equiv 1$

$\left(\frac{y}{2}\right)^2 + \left(\frac{x+y}{2\sqrt{3}}\right)^2 = 1$

$\frac{y^2}{4} + \frac{(x+y)^2}{4\times 3} = 1$

$\underline{(x+y)^2 + 3y^2 = 12}$

Q5

(Total 5 marks)

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6. (i) Find

$$\int x e^{4x}\,dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,dx, \quad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

(i) $\int x\, e^{4x}\, dx$

Let $u = x$ $\qquad$ Let $\frac{dv}{dx} = e^{4x}$

$\frac{du}{dx} = 1$ $\qquad$ $\int dv = \int e^{4x}\,dx$

$v = \frac{1}{4}e^{4x}$

$I = uv - \int v \frac{du}{dx}\,dx$ $\qquad$ By parts

$= x\frac{1}{4}e^{4x} - \int \frac{1}{4}e^{4x}\,dx$

$I = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + c$ ✓

(ii) $\int \frac{8}{(2x-1)^3}\,dx$ $\qquad$ Let $u = 2x-1$

$\frac{du}{dx} = 2 \Rightarrow du = 2dx$

$I = 8\int \frac{1}{u^3}\,dx = 8\int u^{-3}\,\frac{1}{2}du$

$= 8 \times \frac{1}{2}\,\frac{u^{-2}}{-2} + c = \frac{-2}{(2x-1)^2} + c$ ✓

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Question 6 continued

(iii) $\frac{dy}{dx} = e^{x} \text{cosec}\, 2y \ \text{cosec}\, y$

$\frac{1}{\text{cosec}\, 2y \ \text{cosec}\, y} dy = e^{x} dx$

$\int \sin 2y \sin y \ dy = \int e^{x} dx$

$\int (2 \sin y \cos y) \sin y \, dy = \int e^{x} dx$

$\int 2 \sin^2 y \cos y \ dy = \int e^{x} dx$

$2\int \cos y (\sin y)^2 dy = e^{x} + c$

$\frac{2 \times 1}{2+1} (\sin y)^3 = e^{x} + c'$

Using:

$\int f'(x)[f(x)]^n = \frac{1}{n+1}[f(x)]^{n+1} + c$

$\therefore \frac{2}{3} \sin^3 y = e^{x} + c$

Condition: $y = \pi/6$ at $x = 0$

$\frac{2}{3} \sin^3 \pi/6 = e^{0} + c$

$\frac{2}{3} \left(\frac{1}{2}\right)^3 = 1 + c \Rightarrow c = -11/12$

$\therefore \frac{2}{3} \sin^3 y = e^{x} - 11/12$

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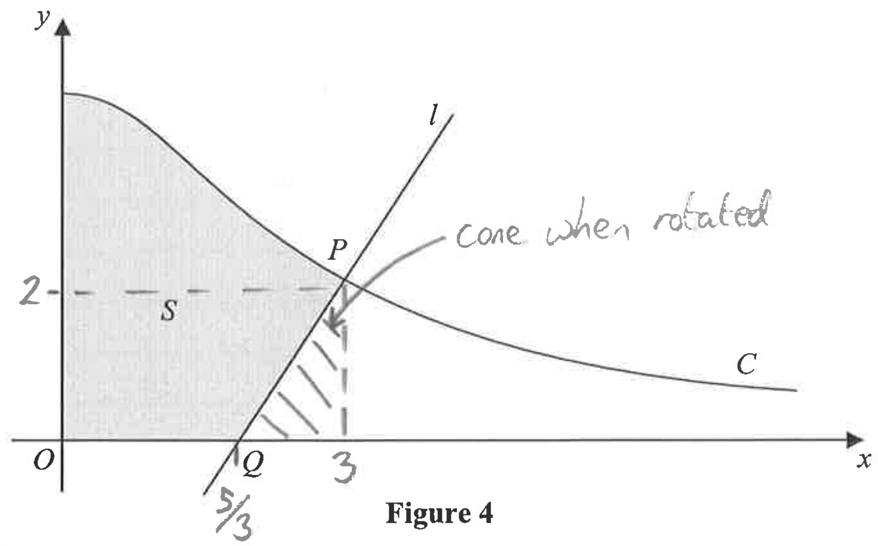

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates (3, 2).

The line l is the normal to C at P. The normal cuts the x-axis at the point Q.

(a) Find the x coordinate of the point Q.

(6)

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis, the y-axis and the line l. This shaded region is rotated 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where p and q are rational numbers to be determined.

[*You may use the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(9)

a) $x = 3\tan\theta$ $\qquad$ $y = 4\cos^2\theta$

$\frac{dx}{d\theta} = 3\sec^2\theta$ $\qquad$ Let $u = \cos\theta$ $\qquad$ $y = 4u^2$

$\frac{du}{d\theta} = -\sin\theta$ $\qquad$ $\frac{dy}{du} = 8u$

Chain rule: $\qquad$ $\therefore dy/d\theta = -8\sin\theta\cos\theta$

$\frac{dy}{dx} = dy/d\theta \times \frac{d\theta}{dx} = \frac{-8\sin\theta\cos\theta}{3\sec^2\theta}$

$dy/dx = -\frac{8}{3}\sin\theta\cos^3\theta$ $\qquad$ gradient of tangent

✓

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Question 7 continued

At $P(3,2)$ $\theta = \tan^{-1} \frac{x}{3} = \tan^{-1} \frac{3}{3} = \frac{\pi}{4}$

At P: $\frac{dy}{dx} = -\frac{8}{3} \sin\frac{\pi}{4} \cos^3\frac{\pi}{4}$

$= -\frac{8}{3} \frac{\sqrt{2}}{2} \left(\frac{\sqrt{2}}{2}\right)^3 = -\frac{2}{3} = m_T$

Normal: $m_N = -1/m_T$

$m_N = +\frac{3}{2}$

Normal: $y = \frac{3}{2}x + c$

At P: $(2) = \frac{3}{2}(3) + c \Rightarrow c = -\frac{5}{4}$

$\therefore y = \frac{3}{2}x - \frac{5}{4}$

At Q: $y = \frac{3}{2}x - \frac{5}{4} = 0 \Rightarrow x = \frac{5}{3}$

b) Volume $= \pi\int_a^b y^2\,dx = \pi\int_{\theta_1}^{\theta_2} y^2 \frac{dx}{d\theta}\,d\theta$

$= \pi\int_0^{\pi/4} (4\cos^2\theta)^2(3\sec^2\theta)\,d\theta$

$= 48\pi\int_0^{\pi/4} \cos^2\theta\,d\theta = 48\pi\int_0^{\pi/4} \frac{1}{2}(1+\cos 2\theta)\,d\theta$

$= 24\pi\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/4}$

$= 24\pi\left(\frac{\pi}{4} + \frac{1}{2}\sin\frac{\pi}{2} - 0 - 0\right)$

Volume under curve $= 6\pi^2 + 12\pi$

See back page for remainder

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8. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Hence find a vector equation for the line l_1

(1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle PBA is θ,

(c) show that $\cos\theta = \dfrac{1}{3}$

(3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2

(2)

The points C and D both lie on the line l_2

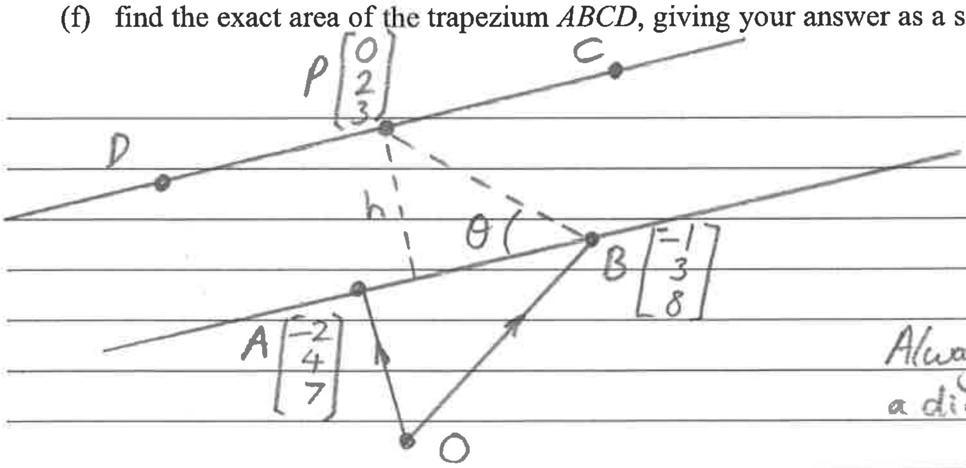
Given that $AB = PC = DP$ and the x coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D.

(3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd.

(4)



Always draw a diagram!

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Question 8 continued

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

b) $\underline{r} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad = \overrightarrow{OA} + \lambda \overrightarrow{AB}$

c) $\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$

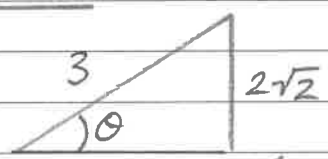
$\overrightarrow{BA} = -\overrightarrow{AB} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

$\cos\theta = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}|\,|\underline{b}|} = \dfrac{\overrightarrow{BA} \cdot \overrightarrow{BP}}{|\overrightarrow{BA}| \cdot |\overrightarrow{BP}|}$

$= \dfrac{(-1 \times 1) + (1 \times -1) + (-1 \times -5)}{\sqrt{1^2 + 1^2 + 1^2}\ \sqrt{1^2 + 1^2 + 5^2}}$

$\cos\theta = \dfrac{3}{\sqrt{3}\,\sqrt{27}} = \dfrac{3}{\sqrt{81}} = \underline{\dfrac{1}{3}}$

d) $l_2 : \underline{r} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$



e) $\overrightarrow{OC} = \overrightarrow{OP} + \overrightarrow{AB} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \overrightarrow{OD} = \overrightarrow{OP} - \overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$

f) Area trapezium $= \frac{1}{2}(a+b)h = \frac{1}{2}(2\overrightarrow{AB} + \overrightarrow{AB})\ \overrightarrow{BP} \sin\theta$

$= \frac{1}{2}(3\sqrt{3})\ \sqrt{3} \times 2\sqrt{2}/3$

$= 9\sqrt{2}$

(Total 15 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END

P 4 3 1 6 5 A 0 2 8 2 8

7. (Cont'd)

Volume cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (2)^2 (3 - 5/3)$

$= \frac{4}{3}\pi \quad 4/3$

$= \frac{16\pi}{9}$ ✓

Volume of S rotated $= V_{curve} - V_{cone}$

$= 6\pi^2 + 12\pi - \frac{16\pi}{9}$

$= 6\pi^2 + \frac{92\pi}{9}$ ✓