Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	9	/	0	1	Signature MODEL ANSWERS	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Friday 24 June 2011 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The curve C has equation $y = 2x^3$, $0 \leqslant x \leqslant 2$.

The curve C is rotated through 2π radians about the x-axis.

Using calculus, find the area of the surface generated, giving your answer to 3 significant figures.

(5)

$y = 2x^3$ Form: $\int f'(x)[f(x)]^n dx = \frac{1}{n+1} f(x)^{n+1} + c$

$\frac{dy}{dx} = 6x^2$

Surface $S = 2\pi \int_{x=0}^{x=2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$= 2\pi \int_0^2 2x^3 \left(1 + (6x^2)^2\right)^{\frac{1}{2}} dx$

$= \frac{4\pi}{36 \times 4} \int_0^2 36 \times 4x^3 (1 + 36x^4)^{\frac{1}{2}} dx$

$= \frac{4\pi}{36 \times 4} \left[\frac{1}{\frac{1}{2}+1} (1 + 36x^4)^{3/2} \right]_0^2$

$\frac{\pi}{36} \left(\frac{2}{3} \left(1 + 36(2)^4\right)^{3/2} \right)$

$S = 806.34$ units^2

$S \simeq 806$ (3 sig fig)

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2. (a) Given that $y = x \arcsin x$, $0 \leqslant x \leqslant 1$, find

(i) an expression for $\dfrac{dy}{dx}$,

(ii) the exact value of $\dfrac{dy}{dx}$ when $x = \dfrac{1}{2}$.

(3)

(b) Given that $y = \arctan(3e^{2x})$, show that

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$$

(5)

a)(i) Let $u = x$ Let $v = \arcsin x$

$\dfrac{du}{dx} = 1$ $\dfrac{dv}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + \arcsin x$$ ✓

(ii) $$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = \frac{\left(\frac{1}{2}\right)}{\sqrt{1-\left(\frac{1}{2}\right)^2}} + \arcsin\left(\tfrac{1}{2}\right)$$

$$= \frac{\sqrt{4}}{2\sqrt{3}} + \frac{\pi}{6} = \frac{\sqrt{3}}{3} + \frac{\pi}{6}$$ ✓

b) Chain rule: $$\frac{dy}{dx} = \frac{6e^{2x}}{1+(3e^{2x})^2} = \frac{6e^{2x}}{1+9e^{4x}}$$

$\times 5$

$2\cosh 2x = e^{2x} + e^{-2x}$

$2\sinh 2x = e^{2x} - e^{-2x}$

$\times 4$

$$= \frac{6}{e^{-2x} + 9e^{2x}}$$ (divide by e^{2x})

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$$ ✓

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3. Show that

(a) $\int_5^8 \frac{1}{x^2 - 10x + 34} \mathrm{d}x = k\pi$, giving the value of the fraction k, **(5)**

(b) $\int_5^8 \frac{1}{\sqrt{(x^2 - 10x + 34)}} \mathrm{d}x = \ln(A + \sqrt{n})$, giving the values of the integers A and n. **(4)**

a) $\int_5^8 \frac{1}{x^2-10x+34}\,dx = \int_0^8 \frac{1}{(x-5)^2+9}\,dx$

Let $u = x-5$ When $x=8$, $u=3$

$du = dx$ $\qquad x=5$, $u=0$

$\therefore I = \int_0^3 \frac{1}{u^2+3^2}\,du = \left[\frac{1}{3}\arctan\frac{u}{3}\right]_0^3$

$= \frac{1}{3}(\arctan 1 - \arctan 0) = \frac{1}{3}(\pi/4 - 0)$

$= \frac{1}{12}\pi \qquad \Rightarrow k = 12$

b) $\int_5^8 \frac{1}{\sqrt{(x-5)^2+9}}\,dx$ Same substitutions as (a)

$\therefore I = \int_0^3 \frac{1}{\sqrt{u^2+3^2}}\,du = \left[\operatorname{arsinh} u/3\right]_0^3$

$= \operatorname{arsinh} 1$

$= \ln(1 + \sqrt{1^2+1})$

$= \ln(1+\sqrt{2})$

Where,
$\operatorname{arsinh} x = \ln(x + \sqrt{x^2+1})$

$A = 1$, $n = 2$

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4.

$$I_n = \int_1^e x^2 (\ln x)^n \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$$

(4)

(b) Find the exact value of I_3.

(4)

a) $I_n = \int_1^e (\ln x)^n x^2 \, dx$

Let $u = (\ln x)^n$ Let $\frac{dv}{dx} = x^2$

$\therefore \frac{du}{dx} = \frac{n}{x}(\ln x)^{n-1}$ $\therefore v = \int x^2 dx = \frac{x^3}{3}$

By parts: $I_n = uv - \int v \frac{du}{dx} dx$

$= (\ln x)^n \frac{x^3}{3} - \int \frac{x^3}{3} \frac{n}{x} (\ln x)^{n-1} dx$

$= \left[\frac{x^3}{3}(\ln x)^n\right]_1^e - \frac{n}{3} \int x^2 (\ln x)^{n-1} dx$

$\underline{I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}}$ ✓

b) $I_3 = \frac{e^3}{3} - \frac{3}{3} I_2 = \cancel{\frac{e^3}{3}} - \left(\cancel{\frac{e^3}{3}} - \frac{2}{3} I_1\right)$

$= \frac{2}{3}\left(\frac{e^3}{3} - \frac{1}{3} I_0\right)$

$= \frac{2e^3}{9} - \frac{2}{9}\left[\frac{x^3}{3} \times 1\right]_1^e$

$I_3 = \frac{2e^3}{9} - \frac{2}{9}\left(\frac{e^3}{3} - \frac{1}{3}\right) = \underline{\frac{4e^3}{27} + \frac{2}{27}}$ ✓

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Question 4 continued

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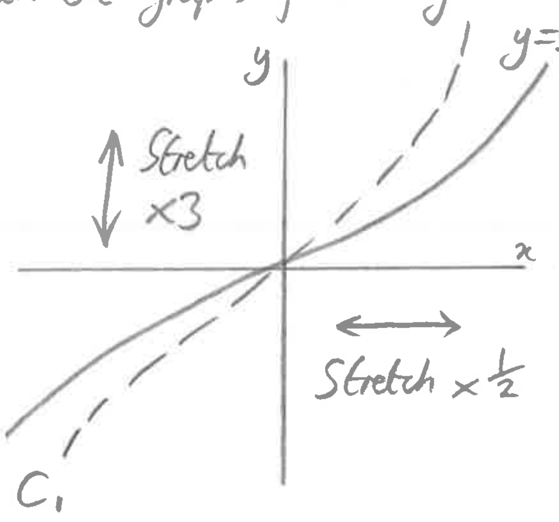
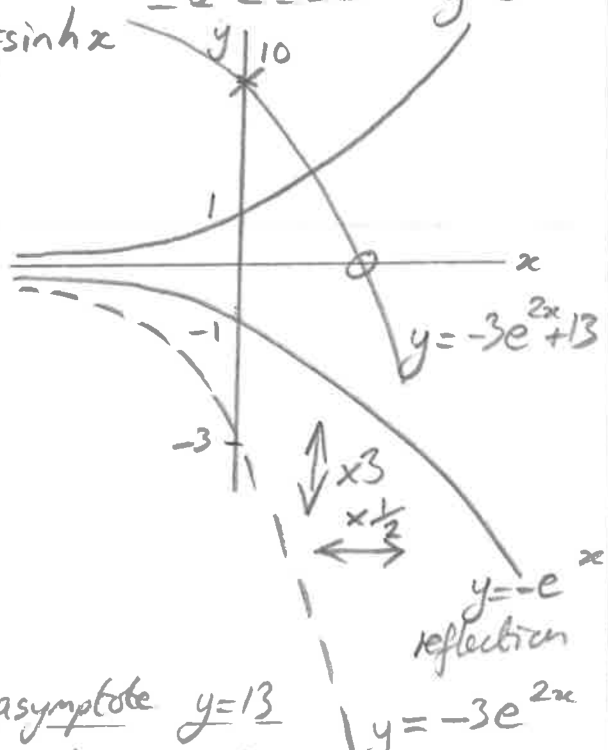
5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.
(4)

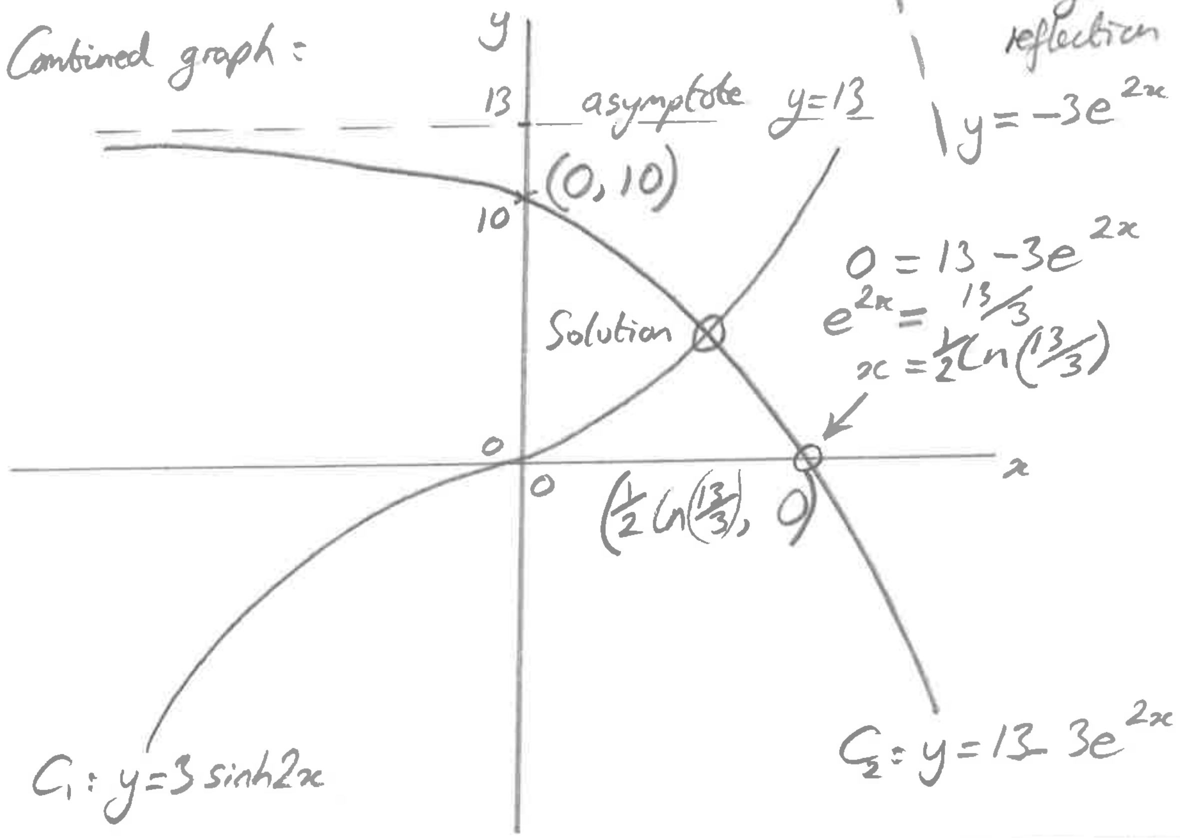
(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer.
(5)

Plan the graphs first: $y = 3\sinh 2x$





Combined graph:



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Question 5 continued

b) $3\sinh 2x = 13 - 3e^{2x}$

$3\left(\frac{e^{2x} - e^{-2x}}{2}\right) = 13 - 3e^{2x}$

$3e^{2x} - 3e^{-2x} = 26 - 6e^{2x}$

$9e^{2x} - 3e^{-2x} - 26 = 0$

$9e^{4x} - 26e^{2x} - 3 = 0 \qquad (\times e^{2x})$

$(9e^{2x} + 1)(e^{2x} - 3) = 0$

Either $9e^{2x} + 1 = 0$

$e^{2x} = -\frac{1}{9}$ reject as invalid

Or $e^{2x} - 3 = 0$

$e^{2x} = 3$

$2x = \ln 3$

$\underline{x = \frac{1}{2}\ln 3} \qquad (k=3)$

✓

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6. The plane P has equation

$$\mathbf{r} = \begin{pmatrix}3\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix} + \mu\begin{pmatrix}3\\2\\2\end{pmatrix} \quad = \underline{a} + \lambda\underline{b} + \mu\underline{c}$$

(a) Find a vector perpendicular to the plane P. **(2)**

The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2).

The acute angle between the plane P and the line l is α.

(b) Find α to the nearest degree. **(4)**

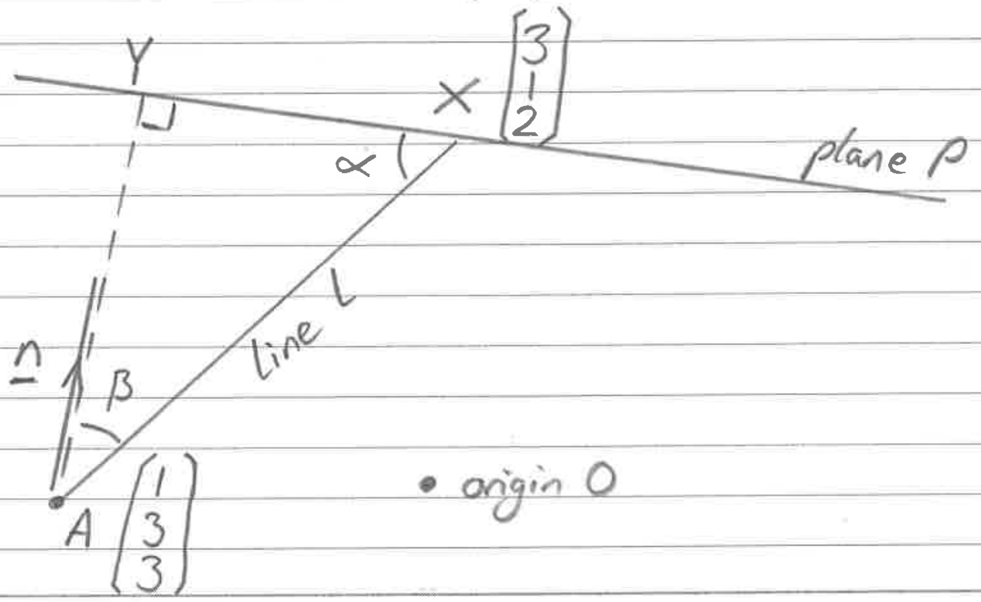
(c) Find the perpendicular distance from A to the plane P. **(4)**

a) $\underline{n} = \underline{b} \times \underline{c}$

$$\underline{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 2 & -1\\ 3 & 2 & 2\end{vmatrix} = \begin{bmatrix}4 - -2\\ -(0 - -3)\\ 0 - 6\end{bmatrix} = \begin{bmatrix}6\\-3\\-6\end{bmatrix}$$

To simplify:

$\underline{n} = t\,\underline{n}' = \begin{bmatrix}2\\-1\\-2\end{bmatrix}$ vector parallel to plane ✓



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Question 6 continued

b) Using $\cos\theta = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|}$

$\cos\beta = \sin\alpha = \dfrac{\underline{n}\cdot\overrightarrow{AX}}{|\underline{n}|\,|\overrightarrow{AX}|}$

$\overrightarrow{AX} = \begin{pmatrix}3\\1\\2\end{pmatrix} - \begin{pmatrix}1\\3\\3\end{pmatrix}$

$= \dfrac{2\times2+(-1)(-2)+(-2)(-1)}{\sqrt{2^2+1^2+2^2}\sqrt{2^2+2^2+1^2}}$

$\overrightarrow{AX} = \begin{pmatrix}2\\-2\\-1\end{pmatrix}$

$\sin\alpha = \dfrac{8}{9}$

$\Rightarrow \underline{\alpha = 63^\circ}$

c) Perpendicular distance to plane $= |\overrightarrow{AY}|$

(see diagram) $= |\overrightarrow{AX}|\sin\alpha$

$= \sqrt{2^2+2^2+1^2}\ \ 8/9$

$= \underline{8/3}$

Leave blank

7. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, \quad k \neq 1$$

(a) Show that det $\mathbf{M} = 2 - 2k$.

(2)

(b) Find $\mathbf{M}^{-1}$, in terms of k.

(5)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}$.

The equation of l_2 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(c) Find a vector equation for the line l_1.

(5)

a) $\det \underline{M} = k(0-2) - (-1)(1--3) + 1(-2-0)$

$= -2k + 4 - 2 = \underline{2-2k}$ ✓

b) Cofactors:

$$\underline{C} = \begin{bmatrix} +(0-2) & -(1--3) & +(-2-0) \\ -(-1--2) & +(k-3) & -(-2k--3) \\ +(1-0) & -(-k-1) & +(0--1) \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} -2 & -4 & -2 \\ -1 & k-3 & 2k-3 \\ 1 & k+1 & 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{bmatrix}$$ transpose

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Question 7 continued

$$\underline{M}^{-1} = \frac{1}{\det \underline{M}} \underline{C}^{T}$$

$$\underline{M}^{-1} = \frac{1}{2-2k} \begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$$

c) $l_2 : (\underline{r} - \underline{a}) \times \underline{b} = 0$ (vector product)

or $\underline{r} = \underline{a} + t\underline{b}$ (vector C4)

$l_2 :$ $\therefore \underline{r} = \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$

If $\underline{l}_2 = \underline{M} \, \underline{l}_1$ mapping

$\therefore \underline{l}_1 = \underline{M}^{-1} \underline{l}_2$ inverse

$$\underline{l}_1 = \frac{1}{2-2(2)} \begin{pmatrix} -2 & -1 & 1 \\ -4 & (2)-3 & (2)+1 \\ -2 & 2(2)-3 & 1 \end{pmatrix} \begin{pmatrix} 4+4t \\ 1+t \\ 7+3t \end{pmatrix}$$

$$\underline{l}_1 = -\frac{1}{2} \begin{pmatrix} -8-8t-1-t+7+3t \\ -16-16t-1-t+21+9t \\ -4-8t+1+t+7+3t \end{pmatrix}$$

$$\underline{l}_1 = -\tfrac{1}{2} \begin{pmatrix} -2-6t \\ 4-8t \\ 0-4t \end{pmatrix} = \begin{pmatrix} +1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$$

Where $\underline{l}_1 = \underline{r}_1$

Leave blank

8. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(a) Use calculus to show that the equation of the tangent to H at the point $(a\cosh\theta, b\sinh\theta)$ may be written in the form

$$xb\cosh\theta - ya\sinh\theta = ab$$

(4)

The line l_1 is the tangent to H at the point $(a\cosh\theta, b\sinh\theta)$, $\theta \neq 0$.
Given that l_1 meets the x-axis at the point P,

(b) find, in terms of a and θ, the coordinates of P.

(2)

The line l_2 is the tangent to H at the point $(a, 0)$.
Given that l_1 and l_2 meet at the point Q,

(c) find, in terms of a, b and θ, the coordinates of Q.

(2)

(d) Show that, as θ varies, the locus of the mid-point of PQ has equation

$$x(4y^2 + b^2) = ab^2$$

(6)

a) Using parametric equations

$x = a\cosh\theta$ $\qquad$ $y = b\sinh\theta$

$dx/d\theta = a\sinh\theta$ $\qquad$ $dy/d\theta = b\cosh\theta$

Chain rule : $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\frac{dy}{dx} = m_T = \frac{b\cosh\theta}{a\sinh\theta}$

Using $y - y_1 = m(x - x_1)$

$y - b\sinh\theta = \frac{b\cosh\theta}{a\sinh\theta}(x - a\cosh\theta)$

Leave blank

Question 8 continued

$$ya\sinh\theta - ab\sinh^2\theta = xb\cosh\theta - ab\cosh^2\theta$$

$$-xb\cosh\theta + ya\sinh\theta = ab\left(\sinh^2\theta - \cosh^2\theta\right)$$

$$xb\cosh\theta - ya\sinh\theta = ab$$

b) At $P\ (x, 0)$

$$xb\cosh\theta - 0 = ab$$

$$x = \frac{a}{\cosh\theta} \quad \text{when } y = 0$$

c) By symmetry and see graph line l_2 is vertical line $x = a$

Sub in $x = a$ into l_1

$$\cancel{a}b\cosh\theta - y\cancel{a}\sinh\theta = \cancel{a}b$$

When $x = a$, $\quad y = \frac{-b}{\sinh\theta} + \frac{b\cosh\theta}{\sinh\theta}$

Useful for part (d) $\quad y = \frac{b(\cosh\theta - 1)}{\sinh\theta}$

d) Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{a}{2\cosh\theta} + \frac{a\cosh\theta}{2\cosh\theta}, \frac{b(\cosh\theta - 1)}{2\sinh\theta}\right)$$

$$= \left(\frac{a(\cosh\theta + 1)}{2\cosh\theta}, \frac{b(\cosh\theta - 1)}{2\sinh\theta}\right)$$

(Total 14 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END

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(8) (d)

$$x = \frac{a(\cosh\theta + 1)}{2\cosh\theta} = \frac{a}{2} + \frac{a}{2\cosh\theta}$$

$$\frac{a}{2\cosh\theta} = x - \frac{a}{2}$$

$$\cosh\theta = \frac{a}{2x-a} \quad (1)$$

This is used to replace / eliminate $\cosh\theta$

$$y = \frac{b(\cosh\theta - 1)}{2\sinh\theta}$$

Sub in (1) ↓

$$\sinh\theta = \frac{b(\cosh\theta - 1)}{2y} \quad (2) \quad = \frac{b\,\cancel{2}(a-x)}{\cancel{2}(2x-a)y} \quad (2A)$$

Identity:

$$\cosh^2\theta - \sinh^2\theta = 1$$

$$\left(\frac{a}{(2x-a)}\right)^2 - \left(\frac{b(a-x)}{(2x-a)y}\right)^2 = 1$$

Sub in (1) & (2A)

$$y^2a^2 - b^2(a-x)^2 = (2x-a)^2y^2$$

$$y^2(4x^2 + \cancel{a^2} - 4xa - \cancel{a^2}) = b^2(a-x)^2$$

$$y^2\,4x\,\cancel{(x-a)} = b^2(a-x)^{\cancel{2}}$$

$$x4y^2 + xb^2 = ab^2$$

$$\underline{x(4y^2 + b^2) = ab^2}$$ ✓

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Further Pure Mathematics FP3 6669
Mark Scheme

Question Number	Scheme	Marks
1.	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2$ and so surface area $=2\pi\int 2x^3\sqrt{(1+(6x^2)^2}\,\mathrm{d}x$	B1
	$=4\pi\left[\frac{2}{3\times 36\times 4}(1+36x^4)^{\frac{3}{2}}\right]$	M1 A1
	Use limits 2 and 0 to give $\frac{4\pi}{216}[13860.016-1]=806$ (to 3 sf)	DM1 A1
		5
	Notes:	
B1	Both bits CAO but condone lack of 2π	
1M1	Integrating $\int\left(y\sqrt{1+\left(\text{their}\frac{dy}{dx}\right)^2}\right)dx$, getting $k(1+36x^4)^{\frac{3}{2}}$, condone lack of 2π If they use a substitution it must be a complete method.	
1A1	CAO	
2DM1	Correct use of 2 and 0 as limits	
2A1	CAO	
2. (a) (i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{\sqrt{(1-x^2)}}+\arcsin x$	M1 A1 (2)
(ii)	At given value derivative $=\frac{1}{\sqrt{3}}+\frac{\pi}{6}=\frac{2\sqrt{3}+\pi}{6}$	B1 (1)
(b)	$\frac{dy}{dx}=\frac{6e^{2x}}{1+9e^{4x}}$	1M1 A1
	$=\frac{6}{e^{-2x}+9e^{2x}}$	2M1
	$=\frac{3}{\frac{5}{2}(e^{2x}+e^{-2x})+\frac{4}{2}(e^{2x}-e^{-2x})}$	3M1
	$\therefore\frac{dy}{dx}=\frac{3}{5\cosh 2x+4\sinh 2x}$ *	A1 cso
		(5) **8**
	Notes:	
(a) M1	Differentiating getting an arcsinx term and a $\frac{1}{\sqrt{1\pm x^2}}$ term	
A1	CAO	
B1	CAO any correct form	

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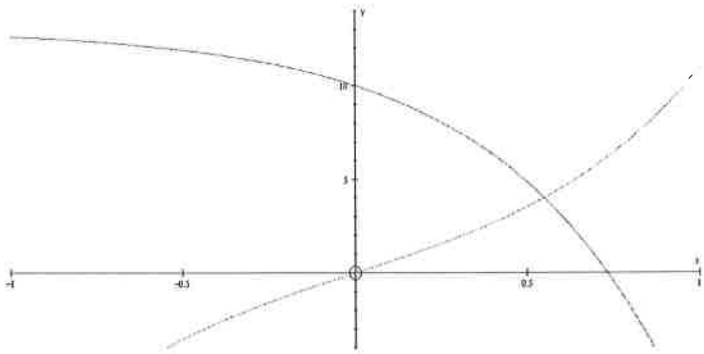
Question Number	Scheme	Marks
(b) 1M1	Of correct form $\dfrac{ae^{2x}}{1\pm be^{4x}}$	
1A1	CAO	
2M1	Getting from expression in e^{4x} to e^{2x} and e^{-2x} only	
3M1	Using sinh2x and cosh2x in terms of $(e^{2x}+e^{-2x})$ and $(e^{2x}-e^{-2x})$	
2A1	CSO – answer given	
3. (a)	$x^2-10x+34=(x-5)^2+9$ so $\dfrac{1}{x^2-10x+34}=\dfrac{1}{(x-5)^2+9}=\dfrac{1}{u^2+9}$	B1
	(mark can be earned in either part (a) or (b))	
	$I=\int\dfrac{1}{u^2+9}du=\left[\dfrac{1}{3}\arctan\left(\dfrac{u}{3}\right)\right]$ | $I=\int\dfrac{1}{(x-5)^2+9}du=\left[\dfrac{1}{3}\arctan\left(\dfrac{x-5}{3}\right)\right]$	M1 A1
	Uses limits 3 and 0 to give $\dfrac{\pi}{12}$ | Uses limits 8 and 5 to give $\dfrac{\pi}{12}$	DM1 A1
		(5)
(b) Alt 1	$I=\ln\left(\left(\dfrac{x-5}{3}\right)+\sqrt{\left(\dfrac{x-5}{3}\right)^2+1}\right)$ or $I=\ln\left(\dfrac{x-5+\sqrt{(x-5)^2+9}}{3}\right)$	M1 A1
	or $I=\ln\left((x-5)+\sqrt{(x-5)^2+9}\right)$	
	Uses limits 5 and 8 to give $\ln(1+\sqrt{2})$.	DM1 A1
		(4)
		9
(b) Alt 2	Uses u = x-5 to get $I=\int\dfrac{1}{\sqrt{u^2+9}}du=\left[\operatorname{ar}\sinh\left(\dfrac{u}{3}\right)\right]=\ln\left\{u+\sqrt{u^2+9}\right\}$	M1 A1
	Uses limits 3 and 0 and ln expression to give $\ln(1+\sqrt{2})$.	DM1 A1
		(4)
(b) Alt 3	Use substitution $x-5=3\tan\theta$, $\dfrac{dx}{d\theta}=3\sec^2\theta$ and so	M1 A1
	$I=\int\sec\theta d\theta=\ln(\sec\theta+\tan\theta)$	
	Uses limits 0 and $\dfrac{\pi}{4}$ to get $\ln(1+\sqrt{2})$.	DM1 A1
		(4)
	Notes:	
(a) B1	CAO allow recovery in (b)	
1M1	Integrating getting k arctan term	
1A1	CAO	
2DM1	Correctly using limits.	
2A1	CAO	

Question Number	Scheme	Marks
(b) 1M1 **1A1** **2DM1** **2A1**	Integrating to get a ln or hyperbolic term CAO Correctly using limits. CAO	
4.		
(a)	$I_n = \left[\frac{x^3}{3}(\ln x)^n\right] - \int \frac{x^3}{3} \times \frac{n(\ln x)^{n-1}}{x} dx$	M1 A1
	$= \left[\frac{x^3}{3}(\ln x)^n\right]_1^e - \int_1^e \frac{nx^2(\ln x)^{n-1}}{3} dx$	DM1
	$\therefore I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$ *	A1cso
		(4)
(b)	$I_0 = \int_1^e x^2 dx = \left[\frac{x^3}{3}\right]_1^e = \frac{e^3}{3} - \frac{1}{3}$ or $I_1 = \frac{e^3}{3} - \frac{1}{3}\left(\frac{e^3}{3} - \frac{1}{3}\right) = \frac{2e^3}{9} + \frac{1}{9}$	M1 A1
	$I_1 = \frac{e^3}{3} - \frac{1}{3} I_0$, $I_2 = \frac{e^3}{3} - \frac{2}{3} I_1$ and $I_3 = \frac{e^3}{3} - \frac{3}{3} I_2$ so $I_3 = \frac{4e^3}{27} + \frac{2}{27}$	M1 A1
		(4) **8**
	Notes:	
(a)1M1 **1A1** **2DM1** **2A1**	Using integration by parts, integrating x^2, differentiating $(\ln x)^n$ CAO Correctly using limits 1 and e CSO answer given	
(b)1M1 **1A1** **2M1** **2A1**	Evaluating I_0 or I_1 by an attempt to integrate something CAO Finding I_3 (also probably I_1 and I_2) If 'n's left in M0 I_3 CAO	

Question Number	Scheme	Marks
5. **(a)**	 Graph of $y = 3\sinh 2x$	B1
	Shape of $-e^{2x}$ graph	B1
	Asymptote: $y = 13$	B1
	Value 10 on y axis and value 0.7 or $\frac{1}{2}\ln\left(\frac{13}{3}\right)$ on x axis	B1
		(4)
(b)	Use definition $\frac{3}{2}(e^{2x} - e^{-2x}) = 13 - 3e^{2x} \to 9e^{4x} - 26e^{2x} - 3 = 0$ to form quadratic	M1 A1
	$\therefore e^{2x} = -\frac{1}{9}$ or 3	DM1 A1
	$\therefore x = \frac{1}{2}\ln(3)$	B1
		(5) **9**
	Notes:	
(a) 1B1	y = 3sinh2x first and third quadrant.	
2B1	Shape of y = $-e^{2x}$ correct intersects on positive axes.	
3B1	**Equation** of asymptote, y = 13, given. Penlise 'extra' asymptotes here	
4B1	Intercepts correct both	
(b) 1M1	Getting a three terms quadratic in e^{2x}	
1A1	Correct three term quadratic	
2DM1	Solving for e^{2x}	
2A1	CAO for e^{2x} condone omission of negative value.	
B1	CAO one answer only	

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Question Number	Scheme	Marks
6. **(a)**	$\mathbf{n} = (2\mathbf{j}-\mathbf{k}) \times (3\mathbf{i}+2\mathbf{j}+2\mathbf{k}) = 6\mathbf{i}-3\mathbf{j}-6\mathbf{k}$ o.a.e. (e.g. $2\mathbf{i}-\mathbf{j}-2\mathbf{k}$)	M1 A1 (2)
(b)	Line l has direction $2\mathbf{i}-2\mathbf{j}-\mathbf{k}$	B1
	Angle between line l and normal is given by $(\cos\beta \text{ or } \sin\alpha) = \frac{4+2+2}{\sqrt{9}\sqrt{9}} = \frac{8}{9}$	M1 A1ft
	$\alpha = 90-\beta = 63$ degrees to nearest degree.	A1 awrt (4)
(c) Alt 1	Plane P has equation $\mathbf{r}.(2\mathbf{i}-\mathbf{j}-2\mathbf{k}) = 1$	M1 A1
	Perpendicular distance is $\frac{1-(-7)}{\sqrt{9}} = \frac{8}{3}$	M1 A1 (4) **10**
(c) Alt 2	Parallel plane through A has equation $\mathbf{r}.\frac{2\mathbf{i}-\mathbf{j}-2\mathbf{k}}{3} = \frac{-7}{3}$	M1 A1
	Plane P has equation $\mathbf{r}.\frac{2\mathbf{i}-\mathbf{j}-2\mathbf{k}}{3} = \frac{1}{3}$	M1
	So O lies between the two and perpendicular distance is $\frac{1}{3}+\frac{7}{3}=\frac{8}{3}$	A1 (4)
(c) Alt 3	Distance A to $(3,1,2) = \sqrt{2^2+2^2+1^2} = 3$	M1A1
	Perpendicular distance is '3' $\sin\alpha = 3\times\frac{8}{9}=\frac{8}{3}$	M1A1 (4)
(c) Alt 4	Finding Cartesian equation of plane P: $2x-y-2z-1=0$	M1 A1
	$d = \frac{\lvert n_1\alpha+n_2\beta+n_3\gamma+d\rvert}{\sqrt{n_1^2+n_2^2+n_3^2}} = \frac{\lvert 2(1)-1(3)-2(3)-1\rvert}{\sqrt{2^2+1^2+2^2}} = \frac{8}{3}$	M1A1 (4)
	Notes:	
(a) M1	Cross product of the correct vectors	
A1	CAO o.e.	
(b) B1	CAO	
M1	Angle between '$2\mathbf{i}-\mathbf{j}-2\mathbf{k}$' and $2\mathbf{i}-2\mathbf{j}-\mathbf{k}$, formula of correct form	
1A1ft	8/9ft	
2A1	CAO awrt	
(c) 1M1	Eqn of plane using $2\mathbf{i}-\mathbf{j}-2\mathbf{k}$ **or** dist of A from O **or** finding length of AP	
1A1	Correct equation (must have =) **or** A to (3,1,2) = 3	
2M1	Using correct method to find perpendicular distance	
2A1	CAO	

Question Number	Scheme	Marks
7. **(a)**	$\text{Det}\,\mathbf{M}=k(0-2)+1(1+3)+1(-2-0)=-2k+4-2=2-2k$	M1 A1 (2)
(b)	$\mathbf{M}^T=\begin{pmatrix} k & 1 & 3 \\ -1 & 0 & -2 \\ 1 & -1 & 1 \end{pmatrix}$ so cofactors $=\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$	M1
	(-1 A mark for each term wrong) $\mathbf{M}^{-1}=\dfrac{1}{2-2k}\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$	M1 A3 (5)
(c)	Let (x, y, z) be on l_1. Equation of l_2 can be written as $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}=\begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix}+\lambda\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.	B1
	Use $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\mathbf{M}^{-1}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ with $k=2$. i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\dfrac{1}{-2}\begin{pmatrix} -2 & -1 & 1 \\ -4 & -1 & 3 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 4+4\lambda \\ 1+\lambda \\ 7+3\lambda \end{pmatrix}$	M1
	$\therefore\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 3\lambda+1 \\ 4\lambda-2 \\ 2\lambda \end{pmatrix}$	M1 A1
	and so $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$ where $\mathbf{a}=\mathbf{i}-2\mathbf{j}$ and $\mathbf{b}=3\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ or equivalent **or** $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$ where $\mathbf{a}=\mathbf{i}-2\mathbf{j}$ and $\mathbf{b}=3\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ or equivalent	B1ft (5) **12**
	Notes:	
(a) M1	Finding determinant at least one component correct.	
A1	CAO	
(b) 1M1	Finding matrix of cofactors or its transpose	
2M1	Finding inverse matrix, 1/(det) cofactors + transpose	
1A1	At least seven terms correct (so at most 2 incorrect) condone missing det	
2A1	At least eight terms correct (so at most 1 incorrect) condone missing det	
3A1	All nine terms correct, condone missing det	
(c) 1B1	Equation of l_2	
1M1	Using inverse transformation matrix correctly	
2M1	Finding general point in terms of λ.	
A1	CAO for general point in terms of one parameter	
2B1	ft for vector equation of their l_1	

Question Number	Scheme	Marks
8.		
(a)	Uses $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{b\cosh\theta}{a\sinh\theta}$ **or** $\frac{2x}{a^2}-\frac{2yy'}{b^2}=0\to y'=\frac{xb^2}{ya^2}=\frac{b\cosh\theta}{a\sinh\theta}$	M1 A1
	So $y-b\sinh\theta=\frac{b\cosh\theta}{a\sinh\theta}(x-a\cosh\theta)$	M1
	$\therefore ab(\cosh^2\theta-\sinh^2\theta)=xb\cosh\theta-ya\sinh\theta$ and as $(\cosh^2\theta-\sinh^2\theta)=1$	A1cso
	$xb\cosh\theta-ya\sinh\theta=ab$ *	
		(4)
(b)	P is the point $(\frac{a}{\cosh\theta},0)$	M1 A1
		(2)
(c)	l_2 has equation $x=a$ and meets l_1 at Q $(a,\frac{b(\cosh\theta-1)}{\sinh\theta})$	M1 A1
		(2)
(d) Alt 1	The mid point of PQ is given by $X=\frac{a(\cosh\theta+1)}{2\cosh\theta}$, $Y=\frac{b(\cosh\theta-1)}{2\sinh\theta}$	1M1 A1ft
	$4Y^2+b^2=b^2\left(\frac{\cosh^2\theta+1-2\cosh\theta+\sinh^2\theta}{\sinh^2\theta}\right)$	2M1
	$=b^2\left(\frac{2\cosh^2\theta-2\cosh\theta}{\sinh^2\theta}\right)$	3M1
	$X(4Y^2+b^2)=ab^2\left(\frac{(\cosh\theta+1)(\cosh\theta-1)2\cosh\theta}{2\cosh\theta\sinh^2\theta}\right)$	4M1
	Simplify fraction by using $\cosh^2\theta-\sinh^2\theta=1$ to give $x(4y^2+b^2)=ab^2$ *	A1cso
		(6)
(d) Alt 2	First line of solution as before	1M1A1ft
	$4Y^2+b^2=b^2\left(\coth^2\theta+\text{cosech}^2\theta-2\coth\theta\,\text{cosech}\theta+1\right)$	2M1
	$=b^2\left(2\coth^2\theta-2\coth\theta\,\text{cosech}\theta\right)$	3M1
	$X(4Y^2+b^2)=ab^2\left(\coth\theta(\coth\theta-\text{cosech}\theta)\right)(1+\text{sech}\theta))$	4M1
	Simplify expansion by using $\coth^2\theta-\text{cosech}^2\theta=1$ to give $x(4y^2+b^2)=ab^2$*	A1cso
		(6)
		14

Question Number	Scheme	Marks
8.		
(a)1M1	Finding gradient in terms of θ	
1A1	CAO	
2M1	Finding equation of tangent	
2A1	CSO (answer given) look for $\pm(\cosh^2\theta - \sinh^2\theta)$	
(b)M1	Putting $y = 0$ into their tangent	
A1ft	P found, ft for their tangent o.e.	
(c) M1	Putting $x = a$ into their tangent.	
A1	CAO Q found o.e.	
(d)	For Alt 1 and 2	
1M1	Finding expressions, in terms of $\sinh\theta$ and $\cosh\theta$ but must be adding	
1A1	Ft on their P and Q,	
2M1	Finding $4y^2 + b^2$	
3M1	Simplified, factorised, maximum of 2 terms per bracket	
4M1	Finding $x(4y^2 + b^2)$, completely factorised, maximum of 2 terms per bracket	
2A1	CSO	
(d)	For Alts 3, 4 and 5	
1M1	Finding expressions, in terms of $\sinh\theta$ and $\cosh\theta$ but must be adding	
1A1	Ft on their P and Q	
2M1	Getting $\cosh\theta$ in terms of x	
3M1	y or y^2 in terms of $\cosh\theta$ **or** $\sinh\theta$ in terms of x and y	
4M1	Getting equation in terms of x and y only. No square roots.	
2A1	CSO	

Question Number	Scheme		Marks
8(d) Alt 3	$X=\dfrac{a(\cosh\theta+1)}{2\cosh\theta},\quad Y=\dfrac{b(\cosh\theta-1)}{2\sinh\theta}$	As main scheme	1M1 A1ft
	$\cosh\theta=\dfrac{a}{2x-a}$	$\cosh\theta$ in terms of x	2M1
	$\sinh\theta=\dfrac{b(\cosh\theta\text{-}1)}{2y}=\dfrac{b(a-x)}{(2x-a)y}$	$\sinh\theta$ in terms of x and y	3M1
	$\left(\dfrac{a}{2x-a}\right)^2-\left(\dfrac{b(a-x)}{(2x-a)y}\right)^2=1$	Using $\cosh^2\theta$ - $\sinh^2\theta=1$	4M1
	Simplifies to give required equation $\left[y^2 4x(a-x)=b^2(a-x)^2,\ x(4y^2+b^2)=ab^2\right]$		A1cso
			(6)
Alt 4	$X=\dfrac{a(\cosh\theta+1)}{2\cosh\theta},\quad Y=\dfrac{b(\cosh\theta-1)}{2\sinh\theta}$	As main scheme	1M1 A1ft
	$\cosh\theta=\dfrac{a}{2x-a}$	$\cosh\theta$ in terms of x	2M1
	$y^2=\dfrac{b^2(\cosh\theta\text{ - }1)^2}{4(\cosh^2\theta\text{ - }1)}=\dfrac{b^2(\cosh\theta\text{ - }1)}{4(\cosh\theta+1)}$	y^2 in terms of $\cosh\theta$ only	3M1
	$y^2=\dfrac{b^2\left(\dfrac{2a-2x}{2x-a}\right)^2}{4\left(\dfrac{2x}{2x-a}\right)}$ o.e	Forms equation in x and y only	4M1
	Simplifies to give required equation		A1 cso
			(6)
Alt 5	$X=\dfrac{a(\cosh\theta+1)}{2\cosh\theta},\quad Y=\dfrac{b(\cosh\theta-1)}{2\sinh\theta}$	As main scheme	1M1 A1ft
	$\cosh\theta=\dfrac{a}{2x-a}$	$\cosh\theta$ in terms of x	2M1
	$y=\left(\dfrac{b(\cosh\theta\text{ - }1)}{2\sinh\theta}\right)=\left(\dfrac{b(\cosh\theta\text{ - }1)}{2\sqrt{\cosh^2\theta\text{ - }1}}\right)$	y in terms of $\cosh\theta$ only	3M1
	Eliminate $\sqrt{\ }$ and forms equation in x and y		4M1
	Simplifies to give required equation		A1cso