Write your name here

Surname	Other names
MODEL ANSWERS	

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mechanics M2

Advanced/Advanced Subsidiary

Friday 24 January 2014 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WME02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. A particle P of mass 2 kg is moving with velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1} when it receives an impulse. Immediately after the impulse is applied, P has velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

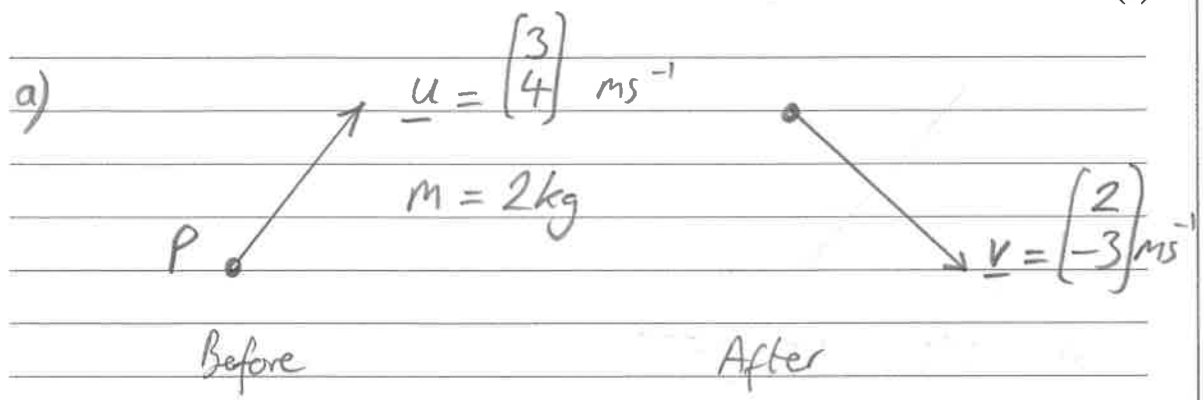
(a) Find the magnitude of the impulse.

(5)

(b) Find the angle between the direction of the impulse and the direction of motion of P immediately before the impulse is applied.

(3)

a)



$$\underline{I} = m\underline{v} - m\underline{u} = m(\underline{v} - \underline{u})$$

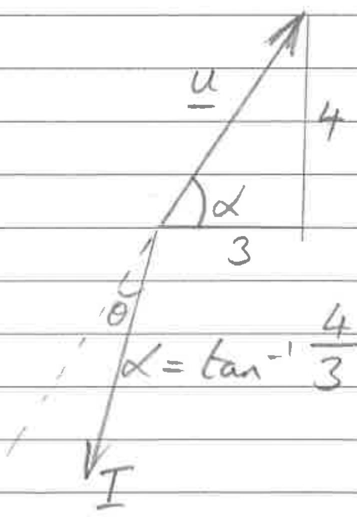
$$= 2\begin{bmatrix} 2-3 \\ -3-4 \end{bmatrix} = \begin{bmatrix} -2 \\ -14 \end{bmatrix}$$

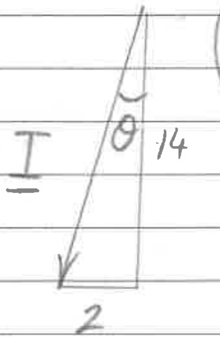
$$|\underline{I}| = \sqrt{2^2 + 14^2} = \sqrt{200} = 10\sqrt{2} \text{ Ns}$$

✓

b)

* Better method: scalar product rule





$\theta = \tan^{-1}\frac{2}{14}$

$\alpha = \tan^{-1}\frac{4}{3}$

$\alpha - \theta = 28.7°$

Angle between $= 180° - 28.7° = 151.3°$

✓

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2. A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\mathrm{m\,s^{-1}}$ in the direction of x increasing, where

$$v = (t-2)(3t-10), \qquad t \geqslant 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P when $t = 3$ **(3)**

(b) Find the total distance travelled by P in the first 3 seconds of its motion. **(6)**

(c) Show that P never returns to O. **(2)**

a) $v = (t-2)(3t-10)$

$v = 3t^2 - 16t + 20$

$a = \frac{dv}{dt} = 6t - 16$

$\left.\frac{dv}{dt}\right|_{t=3} = 6(3) - 16 = \underline{2\ \mathrm{ms^{-2}}}$ ✓

b) $x = \int v\ dt = \int 3t^2 - 16t + 20\ dt$

$x = \frac{3t^3}{3} - \frac{16t^2}{2} + 20t + c$

Apply condition: $x = 0,\ t = 0 \Rightarrow c = 0$

$x(t=3) = 3^3 - 8(3)^2 + 20(3)$

$= 27 - 72 + 60 \ \underline{= 15\ \mathrm{m}}$ PTO.

c) $a = \frac{dv}{dt} = 6t - 16$

Acceleration always +ve after $t = 3$ seconds ✓

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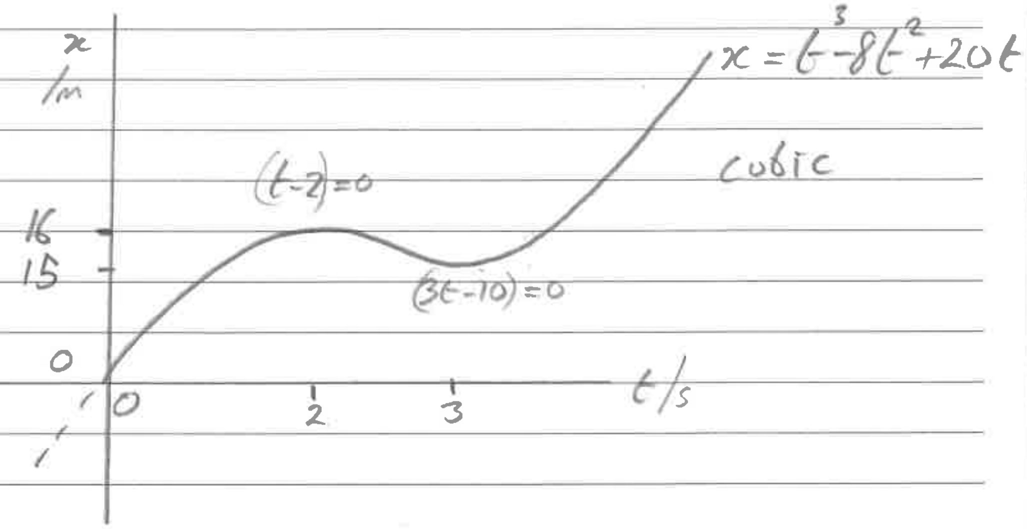
Question 2 continued

b)



When $t=2$, $x = (2)^3 - 8(2)^3 + 20(2) = 16\,m$

$\therefore$ Total Distance travelled $= 16 + (16-15) = 17\,m$
between $t=0$, $t=3$

c) Alternative :

$$x = t^3 - 8t^2 + 20t$$

If $x \neq 0$

$$t^2 - 8t + 20 = 0$$

$$(t-4)^2 + 4 = 0$$

$$t - 4 = \sqrt{-4}$$ so no real solutions

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3. A car has mass 550 kg. When the car travels along a straight horizontal road there is a constant resistance to the motion of magnitude R newtons, the engine of the car is working at a rate of P watts and the car maintains a constant speed of 30 m s^{-1}. When the car travels up a line of greatest slope of a hill which is inclined at θ to the horizontal, where $\sin\theta = \frac{1}{14}$, with the engine working at a rate of P watts, it maintains a constant speed of 25 m s^{-1}. The non-gravitational resistance to motion when the car travels up the hill is a constant force of magnitude R newtons.

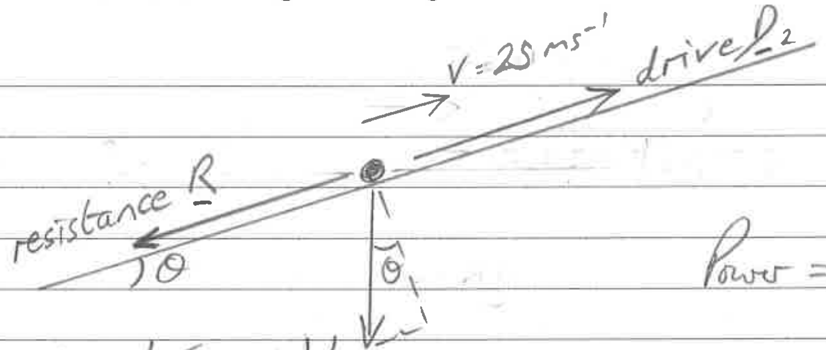
(a) (i) Find the value of R.

(ii) Find the value of P.

(8)

(b) Find the acceleration of the car when it travels along the straight horizontal road at 20 m s^{-1} with the engine working at 50 kW.

(4)



Power = Force × velocity

$\sin\theta = 1/14$ $\quad W = 550g$

Constant speed so

$$F_{res} = ma$$

Slope: $D - R - W_x = 0$

$$\frac{P}{25} - R - \frac{550g}{14} = 0 \qquad ①$$

Flat $\quad \frac{P}{30} - R = 0 \qquad ②$

$$P = 30R \qquad ②A$$

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Question 3 continued

Sub

(2A) into (1)

$$\frac{30R}{25} - R - \frac{550}{14}g = 0$$

$$\frac{5}{25}R = \frac{550}{14} \times 9.8$$

$$R = \frac{550 \times 5 \times 9.8}{14} = \underline{1925N}$$

$$\simeq \underline{1930N}$$

$$\therefore P = 30R = 57750\ N$$

$$\simeq \underline{57800\ N} \quad (3 \text{ sig figs})$$

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4.

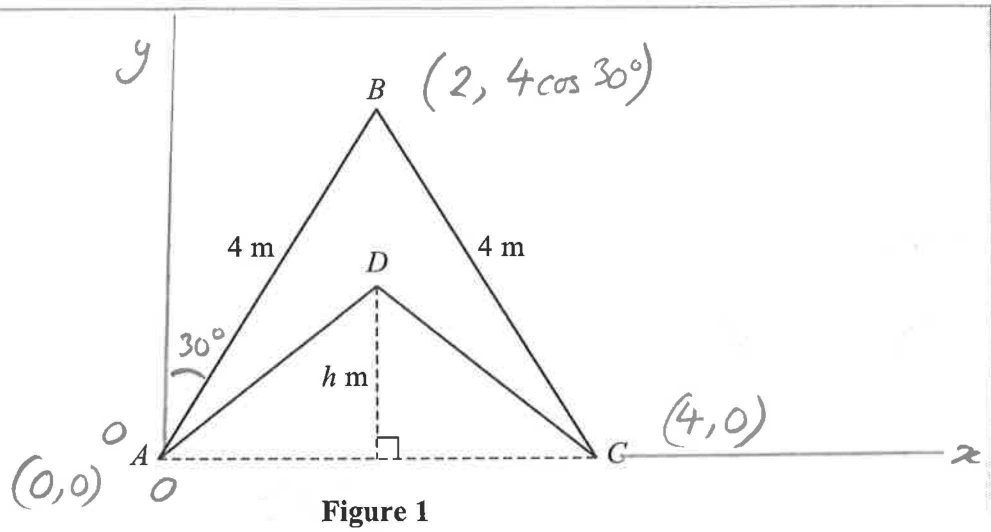

Figure 1

A uniform lamina *ABCD* is formed by removing the isosceles triangle *ADC* of height h metres, where $h < 2\sqrt{3}$, from a uniform lamina *ABC* in the shape of an equilateral triangle of side 4 m, as shown in Figure 1. The centre of mass of *ABCD* is at *D*.

(a) Show that $h = \sqrt{3}$

(7)

The weight of the lamina *ABCD* is W newtons. The lamina is freely suspended from *A*. A horizontal force of magnitude F newtons is applied at *B* so that the lamina is in equilibrium with *AB* vertical. The horizontal force acts in the vertical plane containing the lamina.

(b) Find F in terms of W.

(4)

a) Coordinates of centre of mass of ABC :

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{0 + 2 + 4}{3}, \frac{0 + 0 + 4\cos 30^\circ}{3}\right)$$

$$\text{CofM}_{ABC} = \left(2, \frac{4}{3}\frac{\sqrt{3}}{2}\right) = \left(2, \frac{2\sqrt{3}}{3}\right) \checkmark$$

$$\text{CofM}_{ADC} = \left(\frac{0 + 2 + 4}{3}, \frac{0 + 0 + h}{3}\right) = \left(2, \frac{h}{3}\right) \checkmark$$

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Question 4 continued

Shape	Mass	$\bar{x}$	$\bar{y}$	$m\bar{x}$	$m\bar{y}$
ABC	$\frac{1}{2}(4)(4\cos 30°)$	2	$2\sqrt{3}/3$	$(4\sqrt{3})(2)$	$(4\sqrt{3})\,2\sqrt{3}/3$
ADC	$\frac{1}{2}(4)(h)$	2	$h/3$	$(2h)(2)$	$(2h)\,h/3$

$\Sigma M = 4\sqrt{3} - 2h$

y direction: $\dfrac{m\bar{y}}{\Sigma m} = \dfrac{8 - 2h^2/3}{4\sqrt{3} - 2h} = h$

$8 - 2h^2/3 = 4\sqrt{3}h - 2h^2$ ✓

$\dfrac{4h^2}{3} - 4\sqrt{3}h + 8 = 0$

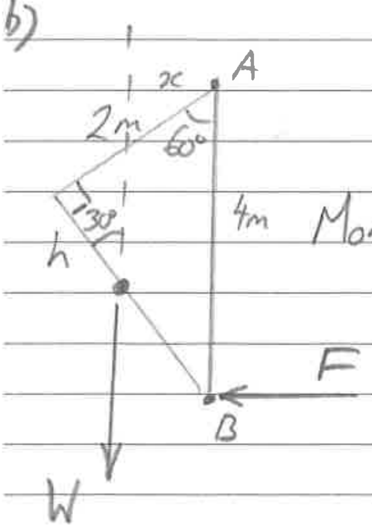
$h^2 - 3\sqrt{3}h + 6 = 0$

$h = \dfrac{3\sqrt{3} \pm \sqrt{(3\sqrt{3})^2 - 4(1)(6)}}{2(1)}$

$h = \dfrac{3\sqrt{3} \pm \sqrt{3}}{2} = \underline{\sqrt{3}}$ ✓

reject $2\sqrt{3}$ as out of range.

b)



Moments A ↻: $F \times 4\text{m} - Wx = 0$

$F = W\left(\dfrac{2\sin 60° - h\sin 30°}{4}\right)$

$\underline{F = W\sqrt{3}/8}$ ✓

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5.

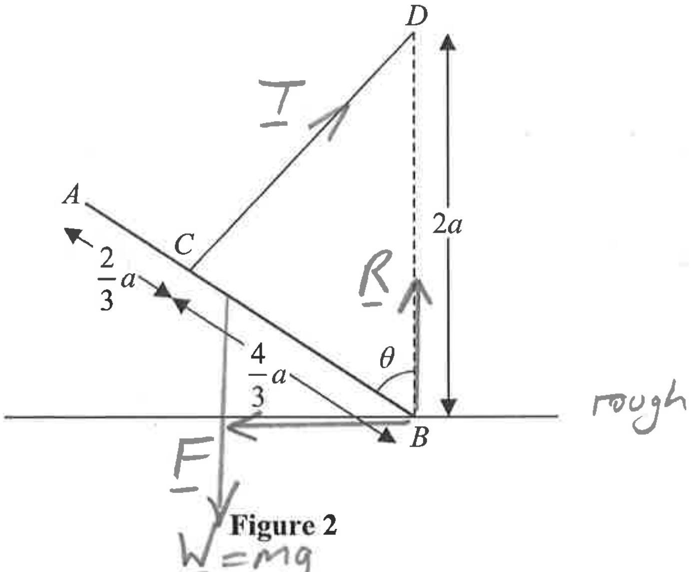

Figure 2

Figure 2 shows a uniform rod AB, of mass m and length $2a$, with the end B resting on rough horizontal ground. The rod is held in equilibrium at an angle θ to the vertical by a light inextensible string. One end of the string is attached to the rod at the point C, where $AC = \frac{2}{3}a$. The other end of the string is attached to the point D, which is vertically above B, where $BD = 2a$.

(a) By taking moments about D, show that the magnitude of the frictional force acting on the rod at B is $\frac{1}{2}mg\sin\theta$ **(3)**

(b) Find the magnitude of the normal reaction on the rod at B. **(5)**

The rod is in limiting equilibrium when $\tan\theta = \frac{4}{3}$

(c) Find the coefficient of friction between the rod and the ground. **(3)**

a) Moments D (↺) $F \times 2a - W \times a\sin\theta = 0$

$\Rightarrow F = \frac{1}{2}mg\sin\theta$ ✓

b) Moments C (↺) $F \times \frac{4}{3}a\cos\theta + W(a - \frac{2}{3}a)\sin\theta - R(2a - \frac{2}{3}a)\sin\theta = 0$ ✓

$\frac{1}{2}mg\frac{4}{3}\cos\theta + W\frac{1}{3} - R\frac{4}{3} = 0$ $(\div\sin\theta)$

$4R = 2mg\cos\theta + mg$ $(\times 3)$

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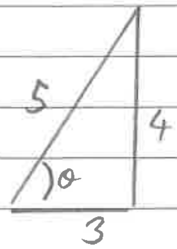
Question 5 continued

$$R = \frac{mg(1 + 2\cos\theta)}{4}$$

c) $\tan\theta = 4/3$



$F_{MAX} = \mu R$

$$\mu = F/R$$

$$= \frac{\frac{1}{2}mg\sin\theta}{\frac{1}{4}mg(1 + 2\cos\theta)}$$

$$= \frac{2\sin\theta}{1 + 2\cos\theta}$$

$$= \frac{2(4/5)}{1 + 2(3/5)}$$

$$= \frac{8}{5} \times \frac{5}{11}$$

$$\mu = 8/11$$

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6. [*In this question the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

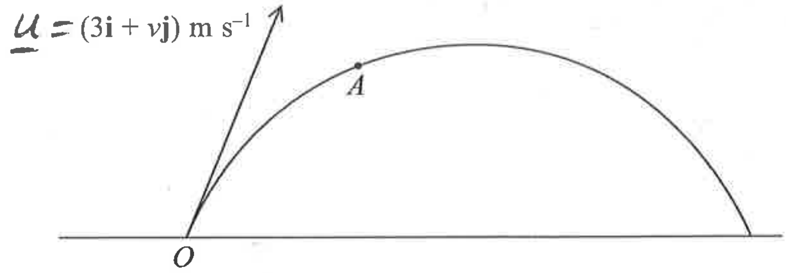

Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(3\mathbf{i} + v\mathbf{j})$ m s^{-1}, $v > 3$. The ball moves freely under gravity and passes through the point A before reaching its maximum height above the horizontal plane, as shown in Figure 3.

The ball passes through A at time $\frac{15}{49}$ s after projection. The initial kinetic energy of the ball is E joules. When the ball is at A it has kinetic energy $\frac{1}{2}E$ joules.

(a) Find the value of v.

(8)

At another point B on the path of the ball the kinetic energy is also $\frac{1}{2}E$ joules.

The ball passes through B at time T seconds after projection.

(b) Find the value of T.

(3)

At O: $E = \frac{1}{2}m|\underline{u}|^2 = \frac{1}{2}m(3^2+v^2)$

At A: $E_k = \frac{1}{2}E = \frac{1}{4}m(3^2+v^2)$

Horizontally: $s = ut$

At A: $S_{Ax} = 3 \times \frac{15}{49}$ m

Vertically: $v = u + at$

At A: $V_{Ay} = v - 9.8 \times \frac{15}{49} = v - 3$

$V_{Ax} = u_x = 3$

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Question 6 continued

At A: $E_k = \frac{1}{2} m v_A^2 \qquad = \frac{1}{4} m (3^2 + v^2)$

$= \frac{1}{2} \cancel{m} ((v-3)^2 + 3^2) = \frac{1}{4} \cancel{m} (3^2 + v^2)$

$= 2(v^2 + 9 - 6v + 9) = 3^2 + v^2$

$2v^2 - v^2 - 12v + 27 = 0$

$v^2 - 12v + 27 = 0$

$(v-9)(v-3) = 0$

$\Rightarrow \underline{v = 9 \text{ ms}^{-1}}$ (reject $v = 3$) ✓

Leave blank

7. Three particles A, B and C, each of mass m, lie at rest in a straight line L on a smooth horizontal surface, with B between A and C. Particles A and B are projected directly towards each other with speeds $5u$ and $4u$ respectively. Particle C is projected directly away from B with speed $3u$. In the subsequent motion, A, B and C move along L. Particles A and B collide directly. The coefficient of restitution between A and B is e.

(a) Find (i) the speed of A immediately after the collision,

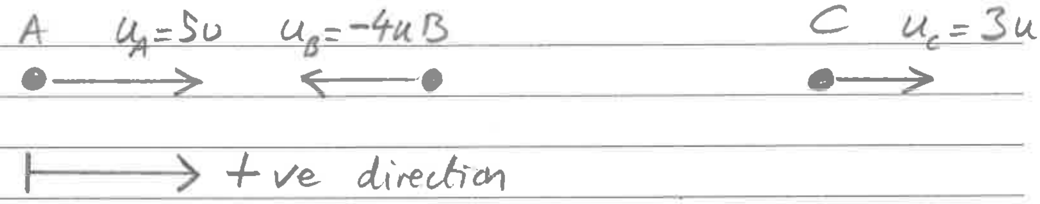
(ii) the speed of B immediately after the collision.

(7)

Given that the direction of motion of A is reversed in the collision between A and B, and that there is no collision between B and C,

(b) find the set of possible values of e.

(4)



a) Coefficient of restitution:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{V_A - V_B}{u_A - u_B}$$

$$e = \frac{V_A - V_B}{5u - -4u} = \frac{V_A - V_B}{9u} \quad (1)$$

Conservation of momentum:

$$mu_A + mu_B = mV_A + mV_B$$

$$5u - 4u = V_A + V_B \quad (2)$$

$$u = V_A + V_B \quad (2A)$$

$$9ue = V_A - V_B \quad (1A)$$

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Question 7 continued

1A + 2A $\quad 2v_A = u(1+9e)$

$|v_A| = |\frac{u}{2}(1+9e)|$

Sub into 2A $\quad v_B = u - \frac{u}{2}(1+9e)$

$|v_B| = |\frac{u}{2}(1-9e)|$ modulus for speed!

b) $v_A = -\frac{u}{2}(1-9e)$ $\leftarrow$ A moves to left in diagram after collision, so

$\therefore \frac{u}{2}(1-9e) < 0$

$e > 1/9$

$\Rightarrow v_A + v_B = 9ue$ 1

$\therefore$ B moves to right.

If B cannot catch & collide with C,

$v_B \leq 3u$

$|\frac{u}{2}(1+9e)| \leq 3u \qquad \cancel{u}9e \leq 6\cancel{u} - \cancel{u}$

$e \leq 5/9$

$\therefore \underline{1/9 < e \leq 5/9}$

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

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