

Write your name here			
Surname MODEL ANSWER	Other names Chris		
Pearson	Centre Number	Candidate Number	
Edexcel GCE			
Statistics S2 Advanced/Advanced Subsidiary			
Monday 27 June 2016 – Morning		Paper Reference	
Time: 1 hour 30 minutes		6684/01	
You must have: Mathematical Formulae and Statistical Tables (Pink)			Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ➤

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PEARSON

1. A student is investigating the numbers of cherries in a *Rays* fruit cake. A random sample of *Rays* fruit cakes is taken and the results are shown in the table below.

x	Number of cherries	0	1	2	3	4	5	≥ 6
f	Frequency	24	37	21	12	4	2	0

- (a) Calculate the mean and the variance of these data. (3)

- (b) Explain why the results in part (a) suggest that a Poisson distribution may be a suitable model for the number of cherries in a *Rays* fruit cake. (1)

The number of cherries in a *Rays* fruit cake follows a Poisson distribution with mean 1.5

A *Rays* fruit cake is to be selected at random.

Find the probability that it contains

- (c) (i) exactly 2 cherries,
(ii) at least 1 cherry. (4)

Rays fruit cakes are sold in packets of 5

- (d) Show that the probability that there are more than 10 cherries, in total, in a randomly selected packet of *Rays* fruit cakes, is 0.1378 correct to 4 decimal places. (3)

Twelve packets of *Rays* fruit cakes are selected at random.

- (e) Find the probability that exactly 3 packets contain more than 10 cherries. (3)

a) Mean:

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{24 \times 0 + 37 \times 1 + 21 \times 2 + 12 \times 3 + 4 \times 4 + 2 \times 5 + 0}{24 + 37 + 21 + 12 + 4 + 2}$$

$$= \frac{141}{100} = 1.41$$

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Question 1 continued

Variance :

$$\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$$

$$= \frac{24 \times 0^2 + 37 \times 1^2 + 21 \times 2^2 + 12 \times 3^2 + 4 \times 4^2 + 2 \times 5^2}{100} - 1.41^2$$

$$= 3.43 - 1.41^2$$

$$= 1.4419 \approx \underline{\underline{1.44}}$$

b) The mean (1.41) and the variance (1.44) are very similar/close.

Therefore, a Poisson distribution is suitable

$$c) X \sim Po(\lambda)$$

$$X \sim Po(1.5)$$

$$(i) P(X=x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

$$P(X=2) = e^{-1.5} \frac{1.5^2}{2!}$$

$$= 0.25102 \dots$$

$$\approx \underline{\underline{0.2510}}$$



Question 1 continued

$$\begin{aligned}
 \text{(ii)} \quad P(X \geq 1) &= 1 - P(X=0) \\
 &= 1 - e^{-1.5} \frac{1.5^0}{0!} \\
 &= 0.7768698\dots \\
 &\approx \underline{0.777}
 \end{aligned}$$

d) Let Y be the random variable for cherries in packets of 5 cakes.

$$Y \sim P_0(1.5 \times 5)$$

$$Y \sim P_0(7.5)$$

$$\begin{aligned}
 P(X > 10) &= P(X \geq 11) \\
 &= 1 - P(Y \leq 10) \\
 &= 1 - 0.8622 \quad \text{Using tables} \\
 &= \underline{0.1378}
 \end{aligned}$$

e) Twelve packets selected at random, same probability of containing more than 10 cherries

$$W \sim B(12, 0.1378)$$

Binomial distribution

- constant probability
- fixed number trials
- trials independent



Question 1 continued

$$P(W=w) = \binom{n}{w} p^w (1-p)^{n-w}$$

$$P(W=3) = \binom{12}{3} 0.1378^3 (1-0.1378)^{12-3}$$

$$= 0.151580 \dots$$

$$\approx \underline{\underline{0.1516}}$$

Q1

(Total 14 marks)



2. In a region of the UK, 5% of people have red hair. In a random sample of size n , taken from this region, the expected number of people with red hair is 3

(a) Calculate the value of n .

(2)

A random sample of 20 people is taken from this region.
Find the probability that

(b) (i) exactly 4 of these people have red hair,

(ii) at least 4 of these people have red hair.

(5)

Patrick claims that *Reddman* people have a probability greater than 5% of having red hair. In a random sample of 50 *Reddman* people, 4 of them have red hair.

(c) Stating your hypotheses clearly, test Patrick's claim. Use a 1% level of significance.

(5)

$$p = 5\% = 0.05$$

$$a) 0.05 n = 3$$

$$n = \frac{3}{0.05}$$

$$\underline{n = 60}$$

b) Fixed sample, $n = 20$

Constant probability, $p = 0.05$

$X \sim B(n, p)$ Binomial distribution

$\sim B(20, 0.05)$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X=4) = \binom{20}{4} 0.05^4 (1-0.05)^{20-4}$$

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Question 2 continued

$$P(X=4) = 0.0133275 \dots$$

$$\approx \underline{0.0133}$$

$$(ii) P(X \geq 4) = 1 - P(X \leq 3)$$

$$= 1 - 0.9841$$

$$= \underline{0.0159}$$

$$c) H_0 : p = 0.05 \quad 1\% \text{ significance.}$$

$$H_1 : p > 0.05 \quad 1\text{-tailed}$$

$$\text{Using } n = 50, \quad p = 0.05$$

$$Y \sim B(n, p)$$

$$Y \sim B(50, 0.05)$$

$$P(Y \geq 4) = 1 - P(Y \leq 3)$$

$$= 1 - 0.7604 \quad \text{Using tables.}$$

$$= \underline{0.2396}$$

Insufficient evidence to reject H_0 .



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Question 2 continued

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Question 2 continued

As $0.2396 > 0.01$, not significant.

There is no evidence to support Patrick's claim.

or

There is no evidence that people in Reddman
have a probability greater than 5% of having
red hair.

↑ "Contextualised" answers needed.

(Total 12 marks)

Q2



3. The random variable R has a continuous uniform distribution over the interval $[5, 9]$

(a) Specify fully the probability density function of R .

(1)

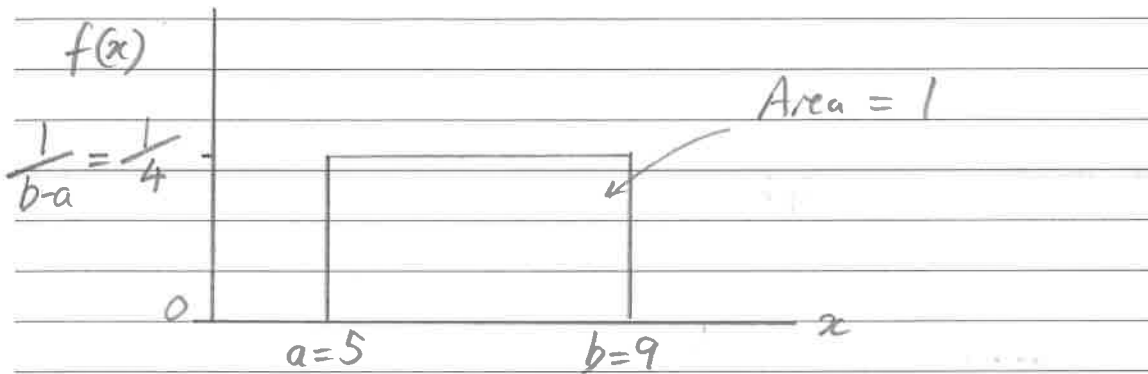
(b) Find $P(7 < R < 10)$

(1)

The random variable A is the area of a circle radius R cm.

(c) Find $E(A)$

(4)



$$a) f(x) = \begin{cases} \frac{1}{4} & , \quad 5 < x < 9 \quad (\text{use } x \text{ or } r) \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$b) P(7 < R < 10) = (9 - 7) \times \frac{1}{4} + 0 \\ = \underline{\underline{\frac{1}{2}}}$$

$$c) A = \pi R^2 \\ \therefore E(A) = E(\pi R^2)$$



Question 3 continued

Using: $\text{Var}(R) = E(R^2) - (E(R))^2$

$$E(R^2) = \text{Var } R + (E(R))^2$$

$$= \frac{(b-a)^2}{12} + \left(\frac{a+b}{2}\right)^2$$

$$= \frac{(9-5)^2}{12} + \left(\frac{5+9}{2}\right)^2$$

$$= \frac{16}{12} + 49$$

$$\underline{E(R^2) = 50\frac{1}{3}}$$

$$\therefore A = \pi E(R^2)$$

$$\underline{A = 50\frac{1}{3}\pi}$$

Q3

(Total 6 marks)



4. A continuous random variable X has cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 2 \\ k(ax + bx^2 - x^3) & 2 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$

Given that the mode of X is $\frac{8}{3}$

- (a) show that $b = 8$

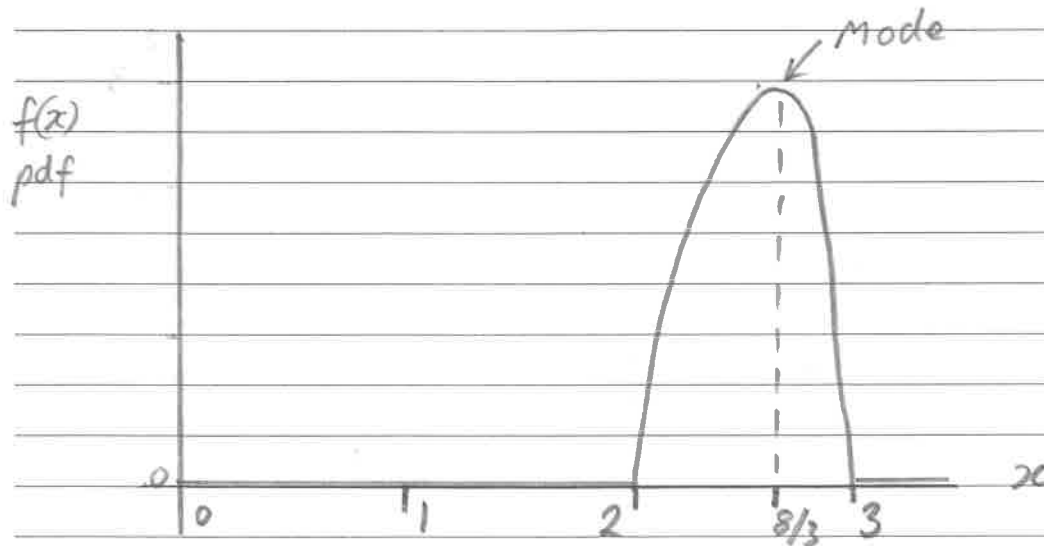
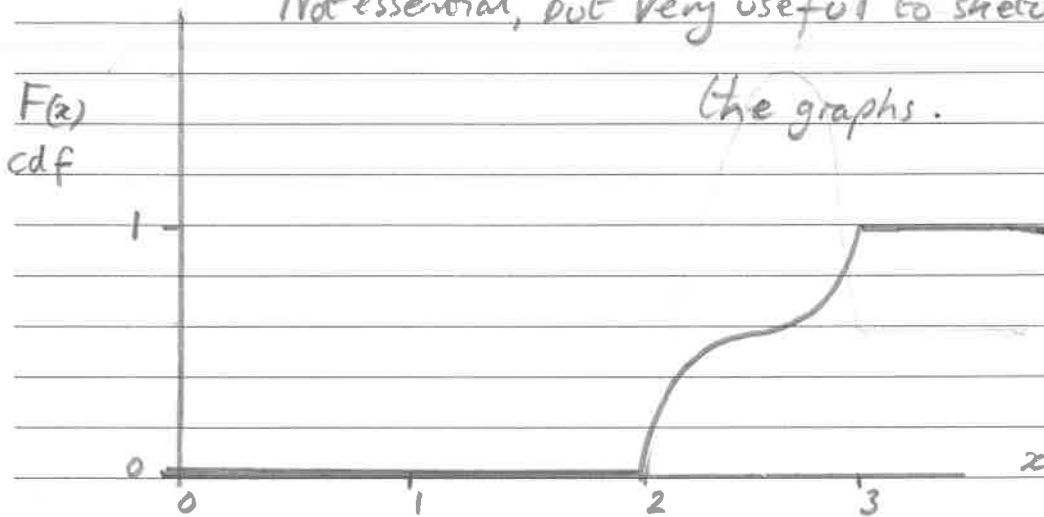
(6)

- (b) find the value of k .

(4)

Not essential, but very useful to sketch

the graphs.



$$f(x) = \frac{d}{dx} F(x) = k(3x^2 + 2bx + a)$$



Question 4 continued

a) Mode occurs when gradient of $f(x)$ is zero,
at a maximum, when $x = \frac{8}{3}$

In range $2 \leq x \leq 3$

$$f(x) = k(-3x^2 + 2bx + a)$$

$$\frac{d}{dx} = k(-6x + 2b)$$

$$\left. \frac{d}{dx} \right|_{x=\frac{8}{3}} = k\left(-6\left(\frac{8}{3}\right) + 2b\right) = 0$$

$$2b = 16$$

$$\underline{b = 8}$$

b) To find a ,

$$F(2) = 0$$

$$F(x) = k(ax + bx^2 - x^3)$$

$$\therefore F(2) = k(a(2) + 8(2)^2 - (2)^3) = 0$$

$$2a + 32 - 8 = 0$$

$$2a = -24$$

$$\underline{a = -12}$$



Question 4 continued

To find k ,

$$F(3) = 1$$

$$\therefore k(-12(3) + 8(3)^2 - (3)^3) = 1$$

$$k(-36 + 72 - 27) = 1$$

$$k 9 = 1$$

$$\underline{k = \frac{1}{9}}$$

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Question 4 continued

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(Total 10 marks)

Q4



P 4 6 6 7 4 A 0 1 5 2 8

5. In a large school, 20% of students own a touch screen laptop. A random sample of n students is chosen from the school. Using a normal approximation, the probability that more than 55 of these n students own a touch screen laptop is 0.0401 correct to 3 significant figures.

Find the value of n .

(8)

X is the random variable representing the number of students who own a touch-screen laptop.

$$X \sim B(n, p)$$

$$X \sim B(n, 0.2)$$

If n is large and p is close to 0.5, then

$$Y \sim N(np, np(1-p)) \quad \text{Normal approximation}$$

$$Y \sim N(0.2n, 0.2n(1-0.2))$$

$$Y \sim N(0.2n, 0.16n)$$

$$P(X > 55) \approx P(Y \geq 55.5) \quad \text{continuity correction applied}$$

$$P(Y \geq 55.5) = 1 - P(Y \leq 55.5)$$

$$0.0401 = 1 - P\left(Z \leq \frac{55.5 - 0.2n}{\sqrt{0.16n}}\right)$$

$$0.9599 = P\left(Z \leq \frac{55.5 - 0.2n}{\sqrt{0.16n}}\right)$$

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Question 5 continued

$$\Rightarrow \frac{55.5 - 0.2n}{\sqrt{0.16n}} = 1.75 \quad \text{using tables in reverse}$$

$$55.5 - 0.2n = 1.75 \times 0.4 \sqrt{n}$$

$$55.5 - 0.2n = \frac{7}{4} \times \frac{4}{10} n^{\frac{1}{2}}$$

$$55.5 - 0.2n = \frac{7}{10} n^{\frac{1}{2}}$$

$$0.2n + 0.7n^{\frac{1}{2}} - 55.5 = 0$$

$$\text{Let } N = n^{\frac{1}{2}}$$

$$2N^2 + 7N - 555 = 0 \quad (\times 10)$$

$$(2N + 37)(N - 15) = 0$$

$$\begin{array}{r} 037 \\ 15 \overline{) 555} \\ \underline{15} \end{array}$$

$$\text{Either } N = \frac{-37}{2} \quad \text{or } N = +15$$

$$\text{reject } \Rightarrow n^{\frac{1}{2}} = 15$$

$$\underline{n = 225}$$



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Question 5 continued

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Q5

(Total 8 marks)



P 4 6 6 7 4 A 0 1 9 2 8

6. A bag contains a large number of counters with one of the numbers 4, 6 or 8 written on each of them in the ratio 5:3:2 respectively.

A random sample of 2 counters is taken from the bag.

$$\frac{5}{10} \quad \frac{3}{10} \quad \frac{2}{10} \quad *$$

- (a) List all the possible samples of size 2 that can be taken.

(2)

The random variable M represents the mean value of the 2 counters.

Given that $P(M=4) = \frac{1}{4}$ and $P(M=8) = \frac{1}{25}$

- (b) find the sampling distribution for M .

(5)

A sample of n sets of 2 counters is taken. The random variable Y represents the number of these n sets that have a mean of 8

- (c) Calculate the minimum value of n such that $P(Y \geq 1) > 0.9$

(3)

a) Combinations of 2 counters:

4, 4	6, 6	8, 8	} Total 6 different pairs
4, 6	-	-	
4, 8	6, 8	-	

b)

mean m	4	5	6	7	8
$P(M=m)$	$\frac{5}{10} \times \frac{5}{10}$	$\frac{5}{10} \times \frac{3}{10} \times 2$	$\frac{3}{10} \times \frac{3}{10} + \frac{5}{10} \times \frac{2}{10} \times 2$	$\frac{3}{10} \times \frac{2}{10} \times 2$	$\frac{1}{25}$
	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{29}{100}$	$\frac{3}{25}$	$\frac{1}{25}$

Sampling distribution $\uparrow$

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Question 6 continued

$$c) Y \sim B(n, p)$$

$$Y \sim B(n, \frac{1}{25})$$

$$P(M=8) = \frac{1}{25}$$

$$P(Y \geq 1) = 1 - P(Y < 1)$$

$$= 1 - \left(\frac{24}{25}\right)^n$$

$$\therefore 1 - P(Y \geq 1) > 0.9$$

$$1 - \left(\frac{24}{25}\right)^n > 0.9$$

$$\left(\frac{24}{25}\right)^n < 0.1$$

Note sign

$$\log \left(\frac{24}{25}\right)^n < \log 0.1$$

Taking logs

$$n \log \left(\frac{24}{25}\right) < \log 0.1$$

$$n > 56.41$$

Note sign change
as $\log \frac{24}{25}$ is negative.

$$\underline{n = 57}$$



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Question 6 continued

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Question 6 continued

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(Total 10 marks)

Q6



P 4 6 6 7 4 A 0 2 3 2 8

7. The weight, X kg, of staples in a bin full of paper has probability density function

$$f(x) = \begin{cases} \frac{9x - 3x^2}{10} & 0 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Use integration to find

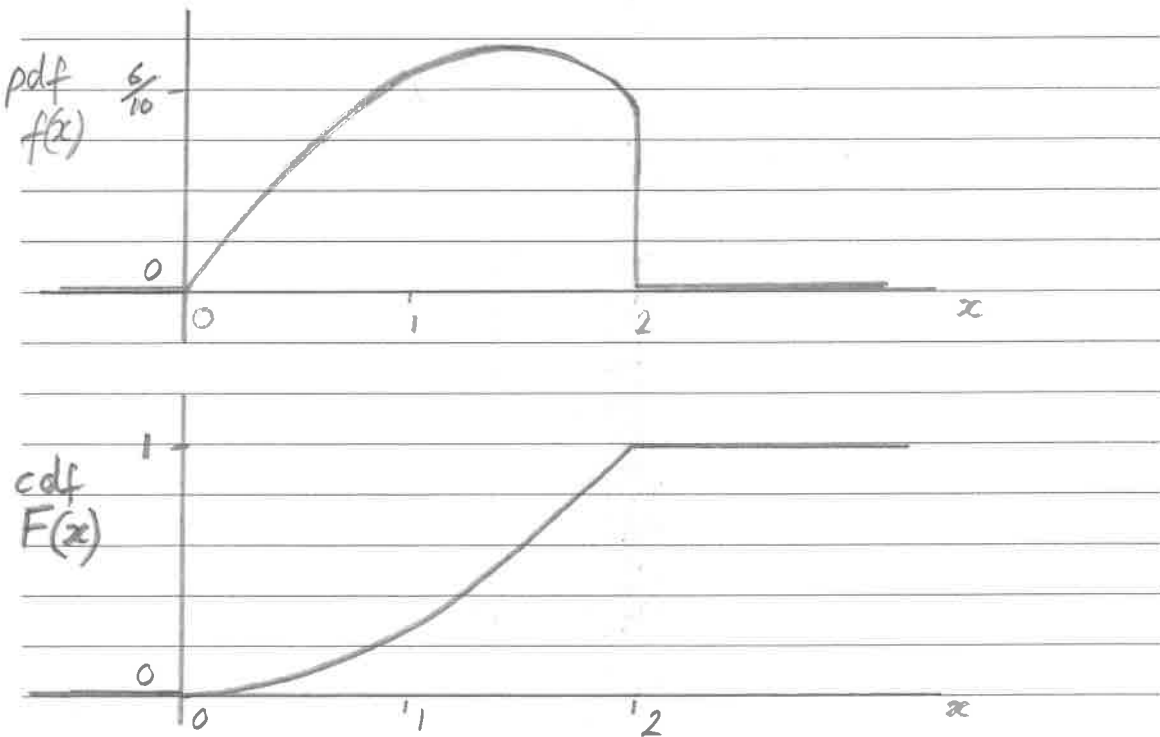
- (a) $E(X)$ (4)
- (b) $\text{Var}(X)$ (4)
- (c) $P(X > 1.5)$ (3)

Peter raises money by collecting paper and selling it for recycling. A bin full of paper is sold for £50 but if the weight of the staples exceeds 1.5 kg it sells for £25

- (d) Find the expected amount of money Peter raises per bin full of paper. (2)

Peter could remove all the staples before the paper is sold but the time taken to remove the staples means that Peter will have 20% fewer bins full of paper to sell.

- (e) Decide whether or not Peter should remove all the staples before selling the bins full of paper. Give a reason for your answer. (2)



Question 7 continued

$$F(x) = \int_{x=-\infty}^{x=\infty} f(x) dx$$

This part is
not required
but interesting!

$$F(2) = \frac{3}{10} \int_{x=0}^{x=2} 3x - x^2 dx$$

But this $F(x)$
is useful for
part (c)
and (d)

$$= \frac{3}{10} \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^2$$

$$= \frac{3}{10} \left(\frac{3(2)^2}{2} - \frac{(2)^3}{3} \right) - 0$$

$$= \frac{3}{10} \left(6 - \frac{8}{3} \right)$$

$$F(2) = \frac{3}{10} \times \frac{10}{3} = \underline{1}$$

$$g) E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_0^2 x \left(\frac{1}{10} (9x - 3x^2) \right) dx$$

$$= \frac{3}{10} \int_0^2 3x^2 - x^3 dx$$



Question 7 continued

$$\begin{aligned}
 E(X) &= \frac{3}{10} \left[\frac{3x^3}{3} - \frac{x^4}{4} \right]_0^2 \\
 &= \frac{3}{10} \left(2^3 - \frac{2^4}{4} \right) - 0 \\
 &= \frac{3}{10} (8 - 4)
 \end{aligned}$$

$$E(X) = \frac{12}{10} = 1.2$$

$$b) \text{Var}(X) = E(X^2) - (E(X))^2$$

$$= \int_{-\infty}^{\infty} x^2 \left(\frac{1}{10} (9x - 3x^2) \right) dx - 1.2^2$$

$$= \frac{3}{10} \int_0^2 3x^3 - x^4 dx - 1.2^2$$

$$= \frac{3}{10} \left[\frac{3x^4}{4} - \frac{x^5}{5} \right]_0^2 - 1.2^2$$

$$= \frac{3}{10} \left(12 - \frac{32}{5} \right) - 1.2^2$$

$$= \frac{84}{50} - \frac{144}{100} = \frac{168 - 144}{100} = \frac{24}{100} = 0.24$$

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Question 7 continued

$$c) F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{10} \left(\frac{3x^2}{2} - \frac{x^3}{3} \right) & 0 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

$$P(X > 1.5) = 1 - P(X < 1.5)$$

$$= 1 - F(1.5)$$

$$= 1 - \left(\frac{3}{10} \left(\frac{3(1.5)^2}{2} - \frac{(1.5)^3}{3} \right) \right)$$

$$= 1 - \frac{3}{10} \times \frac{9}{4} = 1 - \frac{27}{40}$$

$$= \frac{13}{40} = \underline{0.325}$$

$$d) E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= 50 \times (1 - 0.325) + 25 \times 0.325$$

$$= \underline{\pounds 41.875}$$

e) If all staples removed then amount per bin

$$= \pounds 50 \times 0.8 \quad (20\% \text{ fewer bins})$$

= £40. This is less than answer to (d) so he should not remove the staples.



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Question 7 continued

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Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

